PH102, 2013W, Lecture Notes: March 7, Thurs, Lecture 18

Nuclear Reactions refer to any occurrences in which nucleons are changed or exchanged between nuclei. Radioactive decay is a spontaneous nuclear reaction. Nuclear reactions can be induced by striking a nucleus with another nucleus.

- Exothermic nuclear reaction:
    - Q > 0 (kinetic energy is released)
    - total mass decreases after reaction, i.e., $m_i > m_f$
    - Example:

      $${}^{10}_{5}B + {}^{1}_{0}n \rightarrow {}^{7}_{3}Li + {}^{4}_{2}He$$

      Released kinetic energy (Q)

      $$= (10.012937 + 1.008665 - 7.016003 - 4.002603)\, uc^2 = 2.79\, MeV$$

- Endothermic nuclear reaction:
    - Q < 0 (kinetic energy is absorbed)
    - total mass increases after reaction. i.e. $m_i < m_f$
    - Example:

      $${}^{7}_{3}Li + {}^{1}_{1}H \rightarrow {}^{7}_{4}Be + {}^{1}_{0}n$$

      Released kinetic energy (Q)

      $$= (7.016003 + 1.007825 - 7.016928 - 1.008665)uc^2 = -1.64\, MeV$$

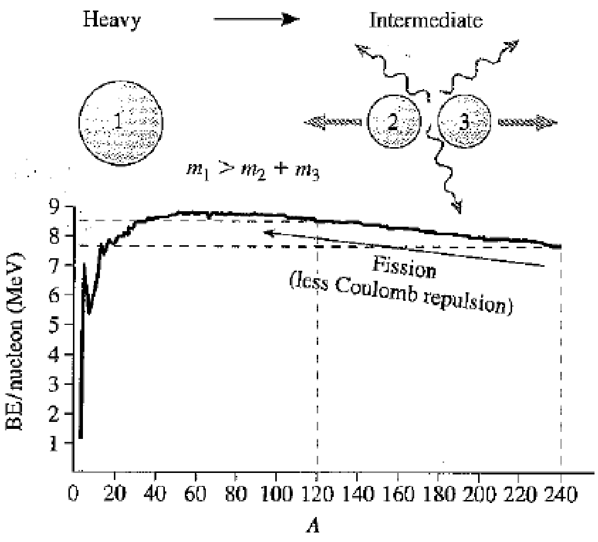
To release energy, mass must decrease after a nuclear reaction, meaning products should be more tightly bound. Considering that the binding energy per nucleon peaks at around A=60, there are two ways to achieve this:

- Nuclear fission: a heavy nucleus breaks into smaller nuclei
- Nuclear fusion: small nuclei fuse together

Nuclear Fission

- A heavy nucleus breaks into two smaller nuclei.
- When this occurs, two or more neutrons are released as soon as small nuclei are formed since large nuclei tend to include more neutrons. Then, subsequent beta decays bring their neutron/proton ratios to stable values.
- The driving force in the binding energy reduction is decrease in Coulomb repulsion.

Figure 11.27 Decreasing BE/nucleon via fission.



- Figure 11.27 shows how much energy can be released per nucleon when a fission occurs from A=240 to A=120. The energy difference is about 0.9 MeV. So, this reaction can potentially generate over 200 MeV when all 240 nucleons are considered in the nucleus of A=240.
- Compare energy release in fission with other energies generated: typical chemical reactions ~ a few eV; spontaneous radioactive decay ~ a few MeV.
- Using the liquid-drop model, the fission process is shown below. A typical excited nucleus can be regarded as an oscillating sphere where surface tension due to strong force and Coulomb repulsions take into play. An excited nucleus can come back to its original sphere shape over time by emitting gamma rays. Sometimes, however, an excited nucleus can be distorted too much (thus Coulomb repulsions win over strong force that provides surface tension) so that the nucleus does not come back to its sphere shape and breaks apart.

FIGURE 24.11 The oscillations of a liquid drop.

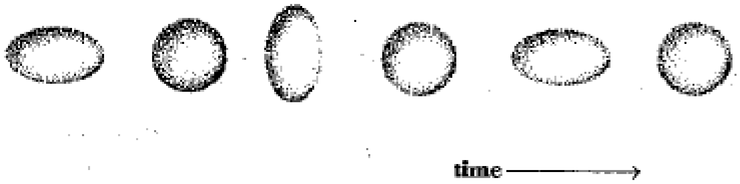


FIGURE 24.12 Nuclear fission according to the liquid-drop model.

- Examples of nuclear fission reactions:

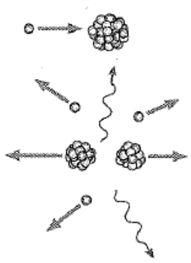
$$^{235}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \rightarrow ^{141}_{56}\mathrm{Ba} + ^{92}_{36}\mathrm{Kr} + 3\,^{1}_{0}\mathrm{n}$$

$$^{235}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \rightarrow ^{140}_{54}\mathrm{Xe} + ^{94}_{38}\mathrm{Sr} + 2\,^{1}_{0}\mathrm{n}$$

$$^{235}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \rightarrow ^{132}_{50}\mathrm{Sn} + ^{101}_{42}\mathrm{Mo} + 3\,^{1}_{0}\mathrm{n}$$

- Since nuclear fission reactions start with a highly energetic neutron striking a large nucleus and highly energetic neutrons are produced as a result of fission reactions, a chain reaction is possible when multiple nucleons are present. See Figure 11.28.

Figure 11.28 Neutron-induced fission, freeing more neutrons.



- Consider that a fission reaction generates E_0 and n neutrons. The first generation of fission would create n neutrons each of which can generate nE_0. The jth generation of fission reactions would create

$$E_j = E_0 n^j$$

- In nature, spontaneous chain reactions of fission do not occur because nuclei that participate in the process are not purified or near one another in mass. For a chain reaction to sustain, a critical assembly of a right size, a right geometry, and purification is needed.

- ${}^{235}_{92}U$ can be engaged in three fission processes. One process generates two neutrons, as compared to three in the other two. Thus the actual energy generated by the nuclear fission of ${}^{235}_{92}U$ can be written as

$$E_j = E_0 k^j$$

Where $k > 1$ exponential energy increase
$k = 1$ controlled energy release
$k < 1$ exponential energy decrease

Nuclear Fusion

- Light nuclei are less tightly bound than those of intermediate mass number. When light nuclei form heavier ones, the total mass decreases and therefore kinetic energy will be released.

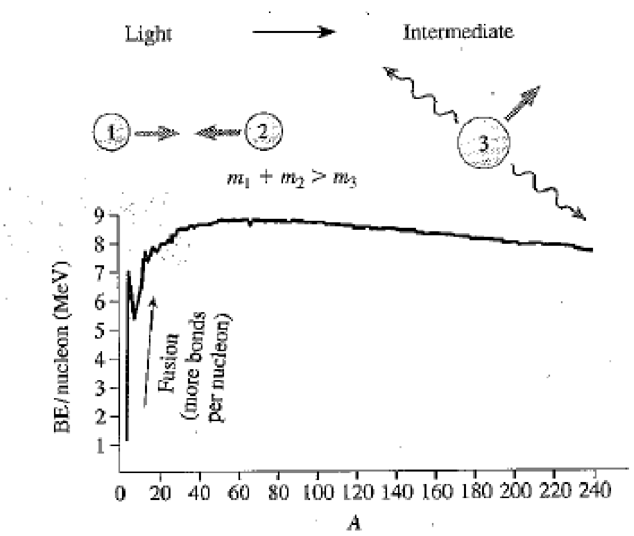
  Example:

$${}^{2}_{1}H + {}^{1}_{1}H \rightarrow {}^{3}_{2}He + \gamma$$

$$Q = (2.0141 + 1.0078 - 3.0160)uc^2$$
$$= 5.48\ MeV$$

- A series of fusion occurs in stars.

Figure 11.30 Decreasing BE/nucleon via fusion.



Proton-Proton Cycle:

Step 1: ${}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + {}_{+1}^{0}\beta + \nu$ (Q=0.42 MeV)
Step 2: ${}^{2}_{1}H + {}^{1}_{1}H \rightarrow {}^{3}_{2}He$ (Q=5.48 MeV)

Step 3: ${}^{3}_{2}He + {}^{3}_{2}He \rightarrow {}^{4}_{2}He + {}^{1}_{1}H + {}^{1}_{1}H$ (Q=12.9 MeV)

Step 1: ${}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + {}_{+1}^{0}\beta + \nu$ (Q=0.42 MeV)
Step 2: ${}^{2}_{1}H + {}^{1}_{1}H \rightarrow {}^{3}_{2}He$ (Q=5.48 MeV)

Net Results: Four protons fuse to ${}^{4}_{2}He$ with Q=24.7 MeV

$${}^{1}_{1}H + {}^{1}_{1}H + {}^{1}_{1}H + {}^{1}_{1}H + {}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{4}_{2}He + {}^{1}_{1}H + {}^{1}_{1}H + 2\,{}_{+1}^{0}\beta + 2\nu$$

Since there is no bound state between two protons, in Step 1, one proton changes into a neutron and emits a positron and a neutrino, which is relatively a slow process involving weak force.

Carbon Cycle: When the temperature and ${}^{4}_{2}He$ concentration is high enough
Two ${}^{4}_{2}He$ will fuse into ${}^{8}_{4}Be$ which is unstable and would naturally decay back to two ${}^{4}_{2}He$. However, if there is an enough number of fast moving ${}^{4}_{2}He$ is available, then a ${}^{8}_{4}Be$ and a ${}^{4}_{2}He$

will fuse to form a ${}^{12}_{6}C$. Once ${}^{12}_{6}C$ appears, more protons will fuse to ${}^{4}_{2}He$ at the faster pace than the Proton-Proton Cycle. The Carbon Cycle is shown below. At even higher temperatures, elements higher than carbon may form. Z higher than 60 may not form by fusion, but neutron capture and subsequent beta- decay may allow Z higher than 60 to form. Heavy elements in the universe are thought to be formed in supernovae.

Step 1: ${}^{1}_{1}H + {}^{12}_{6}C \to {}^{13}_{7}N$
Step 2: ${}^{13}_{7}N \to {}^{13}_{6}C + {}^{0}_{+1}\beta + \nu$
Step 3: ${}^{1}_{1}H + {}^{13}_{6}C \to {}^{14}_{7}N$
Step 4: ${}^{1}_{1}H + {}^{14}_{7}N \to {}^{15}_{8}O$
Step 5: ${}^{15}_{8}O \to {}^{15}_{7}N + {}^{0}_{+1}\beta + \nu$
Step 6: ${}^{1}_{1}H + {}^{15}_{7}N \to {}^{12}_{6}C + {}^{4}_{2}He$

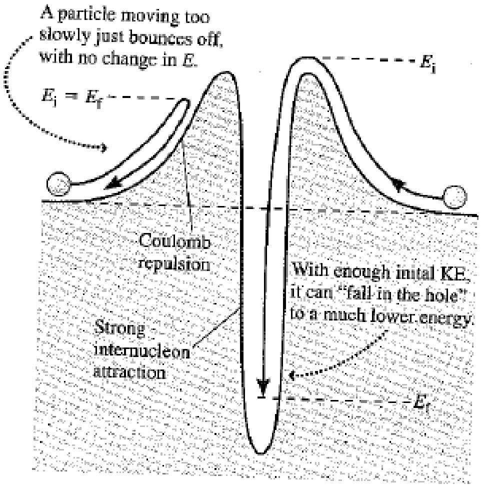
Net effects: 4 protons fuse into a ${}^{4}_{2}He$.

Fission can occur spontaneously, fusion does not occur spontaneously because protons have charges and thus have Coulomb repulsion. Coulomb repulsion should be overcome. To do this, protons should have high initial kinetic energy to get to the lower, bound energy state.

High density and high temperature are necessary to allow more frequent collisions among particles and a greater number of particles have enough energy to surmount the potential barrier.

These conditions can be easily met in stars but not on Earth. Hydrogen bomb uses fusion, but the conditions are set by initially exploding an atomic bomb. Since the energy difference before and after reactions is much greater in fusion than in fission, much higher energy can be released in nuclear fusion reactions.

Figure 11.32 Nuclear fusion: over the Coulomb hurdle, then into the strong force well.



(Power use) Fission and fusion use much smaller amounts of fuel as compared to fossil fuel and provide energy sources that do not involve carbon emission, but produce radioactive byproducts.

	Fission	Fusion
Fuel	Uranium and Thorium: Not rare and should be mined	Deuterium: Abundant and non-toxic $\mathbf{{}^{2}_{1}D + {}^{3}_{1}T \to {}^{4}_{2}He + {}^{1}_{0}n}$ **Q=17.6 MeV**
Waste	Highly toxic Radioactive with long half-life Disposal is a problem	He isotopes (harmless) and Tritium (radioactive with short half life and not chemically hazardous)
Chain reaction	Chain reactions are possible, thus the fission process should be controlled.	Chain reactions are not possible.

Elementary particles

In the modern view of physics, a fundamental force between particles is defined as an exchange of a mediating particle that is specific to that force. Some examples are:

- Gravitational force mediated by graviton (?)
- Electrostatic force mediated by photons
- Weak force mediated by W^+, W^-, and Z^0
- Strong force mediated by Gluon

The mediating particle is called a field quantum, and the study of field quanta in relation to their corresponding fundamental force is called "Quantum field theory."

An analogy for exerting force by exchanging a particle: Snowball exchange

Figure 12.1 A force between students conveyed by exchange of a snowball.

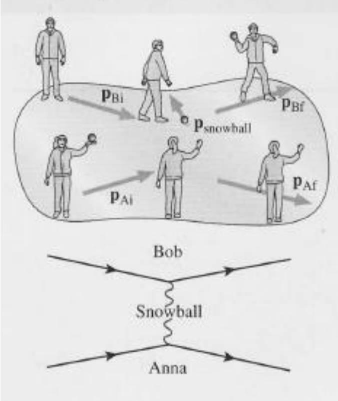


Figure 12.1 illustrates how exchanging a snowball between Bob and Anna can result in a force exerted on both Bob and Anna without direct contact. Anna and Bob are sliding with initial momenta of $\vec{p}_{Bob}\ and\ \vec{p}_{Anna}$ on a frictionless frozen pond. Anna has a snowball and throws it to Bob while sliding. This changes Anna's momentum as well as Bob's momentum as follows:

$$\vec{p}_{Anna(after\ snowball\ throwing)} = \vec{p}_{Anna} - \vec{p}_{snowball}$$

$$\vec{p}_{Bob\ (after\ snowball\ receiving)} = \vec{p}_{Bob} + \vec{p}_{snowball}$$

The force exerted on both Bob and Anna is repulsive as both of them move away from each other. A Feynman diagram is shown this exchange.

This snowball analogy has limitations if it is to apply to fundamental forces:

- It does not show attractive force.
- For fundamental forces, mediating particles only exist during their exchange.
- Mediating particles do not obey the usual momentum and energy rules in the way that real particles do, thus called "virtual particles."

The range of force for mediating particles

- From the uncertainty principle $\Delta t \Delta E \approx \hbar$
- A particle of energy ΔE can exist without violating energy conservation as long as it exists less than $\Delta t \approx \frac{\hbar}{\Delta E}$.
- Consider ΔE is the energy of the mediating particle created and exchanged when the force is exerted, then $\Delta E = mc^2$

$$\Delta t \approx \frac{\hbar}{\Delta E} \approx \frac{\hbar}{mc^2}$$

- Range (Δx) becomes

$$\Delta x \approx c\Delta t \approx \frac{\hbar}{c}\frac{1}{m}$$

From this result, we can see that if a mediating particle has a mass, the range of force is limited. If a mediating particle has no mass, the corresponding force is long ranged. Consider photons in the electromagnetic force.

Antiparticles

- For each kind of particle, there is an antiparticle that shares all the properties of the particle except it is of opposite charge, e.g. electron and positron, proton and antiproton, neutron and antineutron.
- A common notation for an antiparticle is a bar over the notation of particle, e.g. $p\ and\ \bar{p}, n\ and\ \bar{n}$. Sometimes, + and – are used: $e^{-} and\ e^{+}, \mu^{+} and\ \mu^{-}$.
- Particles and their antiparticles can be created together by pair production and can disappear by pair annihilation after producing photon energy equal to their mass energy.
- Antiparticles are detected experimentally.
- Relativistic quantum mechanics provides a theoretical basis for the existence of antiparticles.

Use the following relationship from relativity

$$E = mc^2$$

$$E^2 = m^2c^4 = \frac{{m_0}^2c^4}{1-\frac{v^2}{c^2}} = {m_0}^2c^4\left(\frac{1}{1-\frac{v^2}{c^2}}\right) = {m_0}^2c^4\left(\frac{1}{1-\frac{v^2}{c^2}} - 1 + 1\right)$$

$$= {m_0}^2c^4 + {m_0}^2c^4\left(\frac{\frac{v^2}{c^2}}{1-\frac{v^2}{c^2}}\right) = {m_0}^2c^4 + c^2v^2\frac{{m_0}^2}{1-\frac{v^2}{c^2}} = {m_0}^2c^4 + c^2v^2m^2 = {m_0}^2c^4 + p^2c^2$$

Therefore,

$$E^2 = {m_0}^2c^4 + p^2c^2$$

Using operator notations
$p = -i\hbar\nabla$ and $E = i\hbar\frac{\partial}{\partial t}$

$$E^2\psi = {m_0}^2c^4\psi + p^2c^2\psi$$

$$-c^2\hbar^2\nabla^2\psi + {m_0}^2c^4\psi = -\hbar^2\frac{\partial^2}{\partial t^2}\psi$$

This equation is called the Klein-Gordon equation which can predict the behavior of spinless massive particles at all speeds. For particles with spin, use Dirac Equation where spin is incorporated into the Dirac Equation.

Compare Schrodinger Equation and Klein-Gordon Equation for free particles

	Schrodinger Equation	Klein-Gordon Equation
Energy relation	$E = T$	$E^2 = {m_0}^2c^4 + p^2c^2$
Equation	$-\frac{\hbar^2}{2m}\nabla^2\,\psi = i\hbar\frac{\partial\psi}{\partial t}$	$-c^2\hbar^2\nabla^2\psi + {m_0}^2c^4\psi = -\hbar^2\frac{\partial^2}{\partial t^2}\psi$
Energy	Since $E = \frac{p^2}{2m}$, E is positive	Both positive and negative E values can satisfy the equation.
$\psi^*\psi$	Does not change with time	Changes with time.
Interpretation of $\psi^*\psi$	Probability density of finding a particle in space.	If $\psi^*\psi$ is set in such a way that it does not change with time, then it should be interpreted as Charge density since $\psi^*\psi$ over time remains constant but may be of either charge. The positive energy solution is for a particle while the negative energy solution is for its antiparticle.

It can be considered that the antiparticle state is similar to a hole in a sea of allowed but usually filled negative-energy states of the particle. Figure 12.2 shows that

- the sea of filled antiparticle states will not be detectable.
- A particle can be annihilated when a hole (an antiparticle state) is available (i.e. its antiparticle exists), emitting photon energy of twice the energy of the particle.

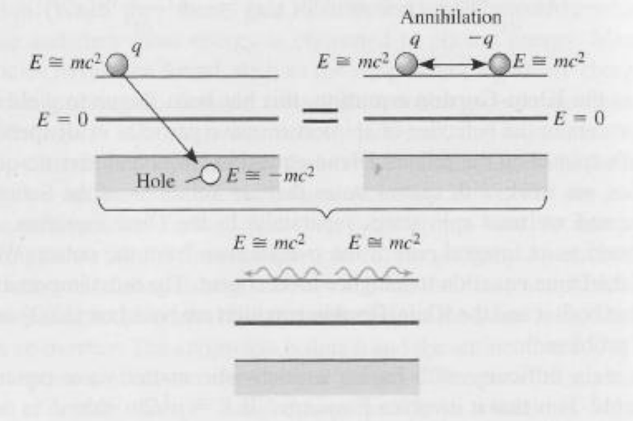


Figure 12.2 Whether a positive-energy particle drops into an empty negative-energy state, or two positive-energy particles annihilate, the result is the same.