

## PH102, 2013W, Lecture Notes: March 5, Tues, Lecture 17

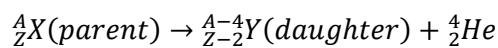
Radioactive decay:

- Definition: An unstable nucleus (parent nucleus) can spontaneously emit small particles or energies to become a nucleus (daughter nucleus) in a more stable state.
- Energy is conserved in radioactive decay:  $Q$  (*kinetic energy released*) =  $(m_i - m_f)c^2$

Three forms of radioactive decay depending upon what is emitted during radioactive decay

### Alpha decay

- Emits an alpha particle (He nucleus=2 protons + 2 neutrons)
- This process makes:



- $Z_{\text{daughter}} = Z_{\text{parent}} - 2$
- $N_{\text{daughter}} = N_{\text{parent}} - 2$
- $A_{\text{daughter}} = A_{\text{parent}} - 4$

- Released kinetic energy ( $Q$ )

$$Q = (m_{\text{parent}} - m_{\text{daughter}} - m_{\alpha}) c^2$$

- Example: alpha decay of  ${}^{238}_{92}\text{U}$

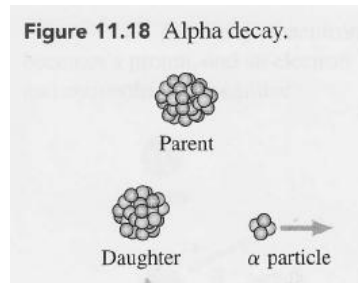
$$\begin{aligned} {}^{238}_{92}\text{U} &\rightarrow {}^{234}_{90}\text{Th} + {}^4_2\text{He} \\ Q &= (m_{\text{parent}} - m_{\text{daughter}} - m_{\alpha}) c^2 \\ &= (238.050784 - 234.043593 - 4.002603)uc^2 \\ &= 0.004588 \times 931.5 \text{ MeV} = 4.27 \text{ MeV} \end{aligned}$$

Since the mass of He is a lot smaller than that of Th, the alpha particle gets almost all the kinetic energy during the alpha decay process.

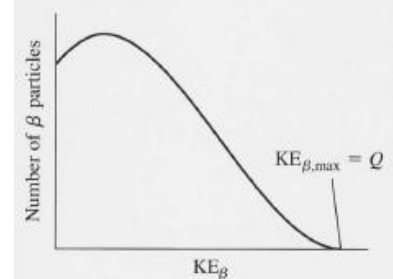
### Beta decay emits temporarily created negatively charged electrons ( $\beta^-$ particle) or positively charged electrons ( $\beta^+$ particle).

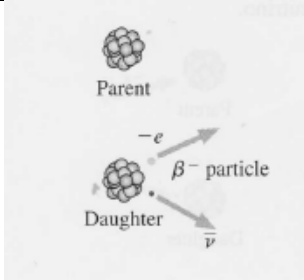
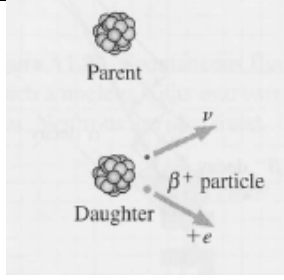
In order to satisfy charge conservation, energy conservation, and angular momentum conservation, a new particle is introduced called neutrinos/antineutrinos.

- Charge conservation: since beta particles carry a charge, the charge of the parent nucleus should increase by 1 in  $\beta^-$  decay and decrease by 1 in  $\beta^+$  decay. As a result, neutrinos' charge should be neutral.
- Angular momentum conservation: since  $\beta$  particles have a spin of  $\frac{1}{2}$ , to conserve angular momentum, neutrinos should have a spin of  $\frac{1}{2}$ .
- Energy conservation. Figure 11.19 shows that kinetic energies of  $\beta$  particles emitted during the decay vary greatly from zero to  $Q$  (maximum allowed). Therefore, neutrinos should carry portions of kinetic energies with  $\beta$  particles. To allow  $\beta$  particles to have the maximum allowed kinetic energy, neutrinos' mass should be negligible.



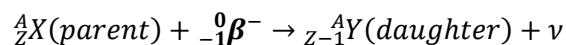
**Figure 11.19** The mysterious variation in  $\beta$  particle energies.



$\beta^-$ decay	$\beta^+$ decay
<ul style="list-style-type: none"> <li>An electron (<math>\beta^-</math> <i>particle</i>) is temporarily created and then emitted.</li> <li>Emits an electron and an anti-neutrino.</li> <li>Changes a neutron inside the nucleus into a proton.</li> </ul>	<ul style="list-style-type: none"> <li>Emits a positron and a neutrino</li> <li>Changes a proton inside the nucleus into a neutron.</li> </ul>
${}^A_Z X(\text{parent}) \rightarrow {}^A_{Z+1} Y(\text{daughter}) + {}^0_{-1} \beta^- + \bar{\nu}$ <ul style="list-style-type: none"> <li><math>Z_{\text{daughter}} = Z_{\text{parent}} + 1</math></li> <li><math>N_{\text{daughter}} = N_{\text{parent}} - 1</math></li> <li><math>A_{\text{daughter}} = A_{\text{parent}}</math></li> </ul> <p>Released kinetic energy</p> $Q = (m_{\text{parent}} - m_{\text{daughter}}) c^2$	${}^A_Z X(\text{parent}) \rightarrow {}^A_{Z-1} Y(\text{daughter}) + {}^0_{+1} \beta^+ + \nu$ <ul style="list-style-type: none"> <li><math>Z_{\text{daughter}} = Z_{\text{parent}} - 1</math></li> <li><math>N_{\text{daughter}} = N_{\text{parent}} + 1</math></li> <li><math>A_{\text{daughter}} = A_{\text{parent}}</math></li> </ul> <p>Released kinetic energy</p> $Q = (m_{\text{parent}} - m_{\text{daughter}} - 2m_{\text{electron}}) c^2$
<p>Example:</p> ${}^{12}_5 B \rightarrow {}^{12}_6 C + {}^0_{-1} \beta^- + \bar{\nu}$ $Q = (12.014352 - 12)uc^2 = 13.4 \text{ MeV}$	${}^{12}_7 N \rightarrow {}^{12}_6 C + {}^0_{+1} \beta^+ + \nu$ $Q = (12.018613 - 12 - 2 \times 0.0005486)uc^2 = 16.3 \text{ MeV}$
	

### The third form of beta decay is Electron Capture.

- A nucleus with too many protons can change a proton into a neutron by capturing an electron.
- Electron capture is easier than  $\beta^+$  decay since an electron is already exists for a nucleus to capture.

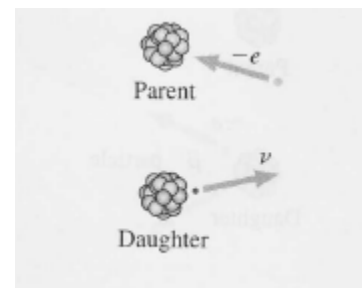
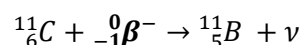


- $Z_{\text{daughter}} = Z_{\text{parent}} - 1$
- $N_{\text{daughter}} = N_{\text{parent}} + 1$
- $A_{\text{daughter}} = A_{\text{parent}}$

- Released kinetic energy

$$Q = (m_{\text{parent}} - m_{\text{daughter}}) c^2$$

- Example:



$$Q = (11.01143 - 11.009305)uc^2 = 1.97 \text{ MeV}$$

### Gamma Decay

- A nucleus in an excited state emits photons (gamma particles,  $\gamma$ ) to go into a lower energy state.
- Gamma decay does not alter N or Z.
- Gamma energies are characteristic of a given isotope, and are thus used to identify the isotope.

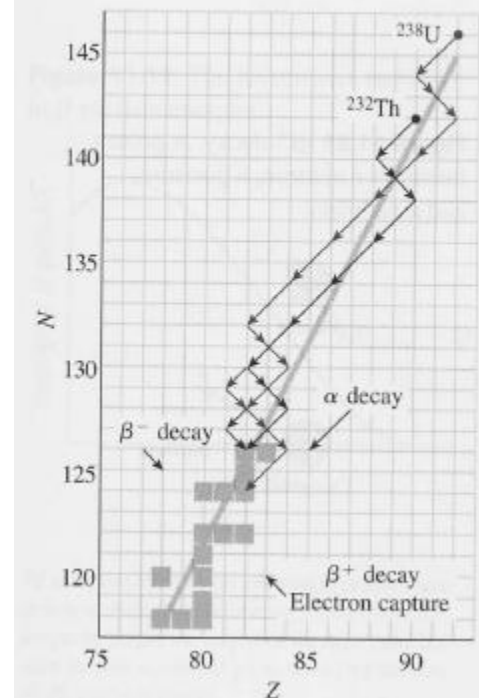
Figure 11.24 Gamma decay.



### Decay series

- An unstable nucleus can be involved in a series of decays until it finds a stable state.
- We can plot this process on a graph that represents N and Z numbers of each nucleus in the series.
- Figure 11.23 shows such a graph. The gray line represents the line of stability. The figure shows two series: one for U-238 and the other for Th-232.
- Alpha decay is shown by an arrow
  - $Z_{daughter} = Z_{parent} - 2$
  - $N_{daughter} = N_{parent} - 2$
- $\beta^-$  decay
  - $Z_{daughter} = Z_{parent} + 1$
  - $N_{daughter} = N_{parent} - 1$
- $\beta^+$  decay and electron capture
  - $Z_{daughter} = Z_{parent} - 1$
  - $N_{daughter} = N_{parent} + 1$

Figure 11.23 The “directions” of  $\alpha$  and  $\beta$  decays, and the decay series of uranium-238 and thorium-232.



### Radioactive Decay Law

- For all decays, the rate of decay over time will be proportional to the sample size:

$$\frac{dN}{dt} \propto N$$

$$\frac{dN}{dt} = -\lambda N \quad \text{where } N = \text{Number of nuclei}; \lambda = \text{decay constant}$$

$$\frac{dN}{N} = -\lambda dt$$

$$\int_{N_0}^N \frac{dN}{N} = -\lambda \int_0^t dt$$

$$\ln \frac{N}{N_0} = -\lambda t$$

$$N = N_0 e^{-\lambda t}$$

- Decay rate  $R = \lambda N$  (decays per second)
- A sample of the same nuclei will decay by the same fraction in equal successive intervals of time.

- Half-life ( $T_{1/2}$ ) is defined as the time interval at which half of the sample will decay:

$$\frac{1}{2}N_0 = N_0 e^{-\lambda T_{1/2}}$$

From this relationship, we can calculate decay constant

$$\lambda = \frac{\ln 2}{T_{1/2}}$$

Figure 11.26 shows the relationship between half life and the number of nuclei remaining over time.

- Half-lives vary widely, from  $10^{-22}$  seconds to  $10^{+17}$  years.

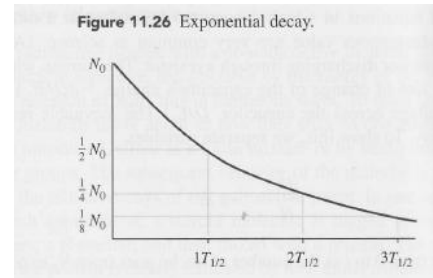


TABLE 11.3 Selected decays

Isotope	Decay Mode	Half-Life
$^{35}_{20}\text{Ca}$	$\beta^+$	50 ms
$^3_1\text{H}$	$\beta^-$	12.3 yr
$^{238}_{92}\text{U}$	$\alpha$	$4.5 \times 10^9$ yr

Example:

A sample holds 2  $\mu\text{g}$  of tritium.

- Initial decay rate (R)

$$R = \lambda N = \frac{\ln 2}{T_{1/2}} \cdot \frac{\text{sample mass}}{\text{atomic mass of Tritium}}$$

Since Tritium's atomic mass = 3.02 u and 1 u =  $1.66 \times 10^{-27}$  kg,  $T_{1/2} = 12.3 \text{ yr} = 12.3 \times 3.16 \times 10^7 \text{ sec}$

$$\lambda = \frac{\ln 2}{T_{1/2}} = 1.78 \times 10^{-9} \text{ /sec}$$

$$R = 7.1 \times 10^8 \text{ decays/sec}$$

- Elapsed time before the decay rate falls to 1% of its initial value?

$$N = N_0 e^{-\lambda t}$$

$$\frac{1}{100} N_0 = N_0 e^{-\lambda t}$$

$$t = \frac{-\ln(\frac{1}{100})}{\lambda} = 2.6 \times 10^9 \text{ sec} = 81.7 \text{ years}$$

### Carbon-14 dating

- Carbon-14's  $\beta^-$  decay has a half-life of 5730 years.
- Carbon-14 dating only works for formerly living organisms.
- Carbon-14's amount is constantly maintained for living organisms since living organisms exchange Carbon with the environment. Ratio of naturally produced C14/C12 =  $1.3 \times 10^{-12}$
- When, a living organism dies, it stops the exchange process, thus C-14 in the dead organism decays exponentially.

Example:

What is the age of a fossil sample that contains 6 g of carbon and has a decay rate (R) of 30 decays per minute?

- The sample has 6 g of Carbon.  $1.3 \times 10^{-12}$  th of 6g of Carbon should be C-14 at the time when the organism in the sample was alive. This amount of C-14 is subject to the decay. Therefore,

$$N_0(C_{14}) = (1.3 \times 10^{-12}) \cdot \left(\frac{6 \text{ g}}{12 \text{ g}}\right) \cdot (6.02 \times 10^{23}) = 3.9 \times 10^{11}$$

- Decay constant

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{5730 \text{ years} \cdot 3.16 \times 10^7 \text{ sec/year}} = 3.83 \times 10^{-12} \text{ /sec}$$

- Current decay rate (R)=30 decays/60 seconds=1/2 decays/sec

$$R = N \cdot \lambda = N \cdot 3.83 \times 10^{-12} \text{ /sec} = 0.5 \text{ /sec}$$

$$N = 1.31 \times 10^{11}$$

$$N = N_0 e^{-\lambda t}$$

$$t = -\frac{1}{\lambda} \ln \frac{N}{N_0} = -\frac{1}{3.83 \times 10^{-12} \text{ /sec}} \ln \frac{1.31 \times 10^{11}}{3.9 \times 10^{11}} = 2.86 \times 10^{11} \text{ sec} \sim 9000 \text{ years}$$