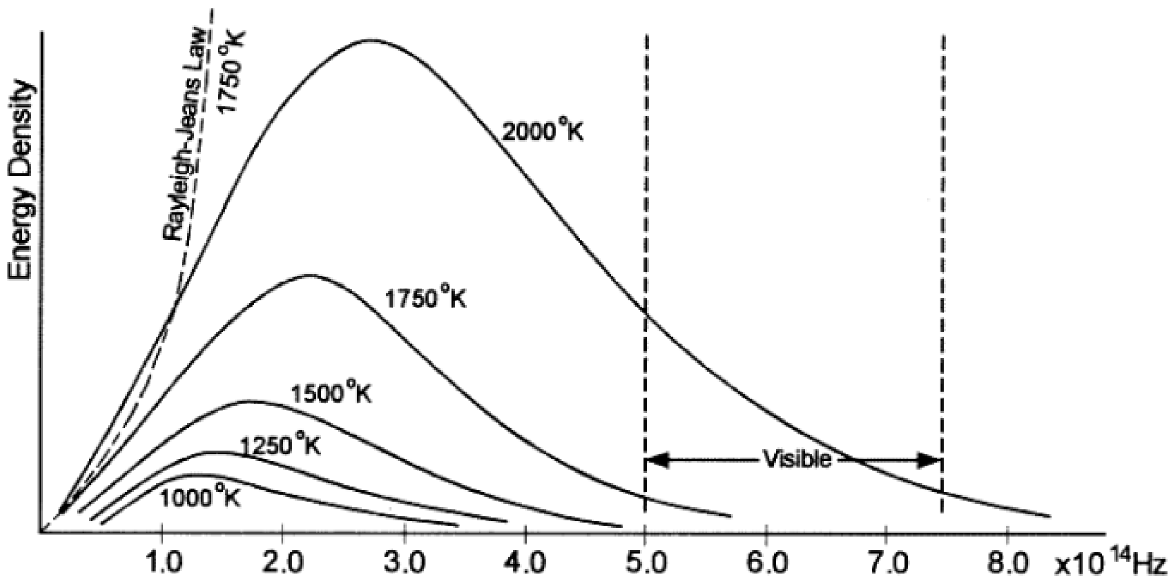
PH102, 2013W, Lecture Notes: February 7, Thur, Class 10

Black Body Radiation

- Every substance emits electromagnetic radiation depending upon the temperature of the substance.
- A "black body" refers to an ideal body that absorbs all radiation incident upon it regardless of frequency and emits the radiation.
- The spectral distribution of energy in the radiation depends only upon the temperature of the body as shown below:

Figure 6.24



- By experimental results, the following two relationships were discovered:
  - Wien's law: the frequency at which the maximum photon energy occurs is proportional to temperature of the black body.
  - Stefan-Boltzmann law: The intensity of radiation $\propto T^4$

Rayleigh-Jeans Approach (Classical approach) that led to ultraviolet catastrophe

Consider a massless photon moving inside a three dimensional cubic box (L x L x L)
For a photon to exist, its frequency or its wavelength should be quantized (to make standing waves) as follows:

$$n_x = \frac{2L}{\lambda} = 1, 2, 3, \ldots$$

$$n_y = \frac{2L}{\lambda} = 1, 2, 3, \ldots$$

$$n_z = \frac{2L}{\lambda} = 1, 2, 3, \dots$$

For a standing wave in any arbitrary direction, it must be true that

$$n^2 = n_x^2 + n_y^2 + n_z^2 = (\frac{2L}{\lambda})^2 \quad where \begin{cases} n_x = 0, 1, 2 \dots \\ n_y = 0, 1, 2 \dots \\ n_z = 0, 1, 2 \dots \\ except\ when\ all\ n_x n_y n_z\ are\ zero \end{cases}$$

$$n = \frac{2L}{\lambda} \rightarrow \quad dn = -\frac{2L}{\lambda^2} d\lambda$$

The umber of permissible states in the three dimensional space can be written as

$$N(n)dn = 2\text{x}\frac{1}{8}4\pi n^2 dn = \pi n^2 dn = \pi \left(\frac{2L}{\lambda}\right)^2 \left(-\frac{2L}{\lambda^2} d\lambda\right) = -\frac{8\pi L^3 d\lambda}{\lambda^4} = -\frac{8\pi V d\lambda}{\lambda^4} \equiv -N(\lambda)d\lambda$$

Note that Rayleigh multiplied by 2 to account for two polarization possibilities. As a result, the actual number of standing waves in the cavity would be

$$\frac{N(\lambda)d\lambda}{V} = \frac{8\pi}{\lambda^4} d\lambda$$

Since

$$\lambda = \frac{c}{\nu} \quad and\ d\lambda = -\frac{c\ d\nu}{\nu^2}$$

The number of permissible states in terms of ν

$$N(n)dn = -\frac{8\pi V d\lambda}{\lambda^4} == -\frac{8\pi V}{(\frac{c}{\nu})^4}\left(-\frac{c\ d\nu}{\nu^2}\right) = \frac{8\pi V}{c^3}\nu^2 d\nu \equiv N(\nu)d\nu$$

Energy associated with a photon would be 2 x 1/2 $k_B T$ because light has both electric and magnetic energy origins (two degrees of freedom) according to the equipartition theorem.

Thus, the energy radiation rate can be expressed in terms of ν:

$$u(\nu)d\nu = k_B T N(\nu)d\nu = \frac{8\pi V k_B T}{c^3}\nu^2 d\nu$$

This expression tells us that the rate of energy radiation is proportional to ν^2. This means that the energy radiation rate can go to infinity as ν goes to infinity! IMPOSSIBLE!! This was known as ultraviolet catastrophe.

The number of permissible states can be rewritten in terms of energy

$$E = h\nu \text{ and } \nu = \frac{E}{h} \text{ and } d\nu = \frac{1}{h} dE$$

The total number of permissible states in terms of E would be

$$N(E)dE = \frac{8\pi V}{c^3 h^3} E^2 dE$$

Therefore, Density of States would be

$$D(E) = \frac{8\pi V}{c^3 h^3} E^2$$

Planck Hypothesis which virtually leads to Bose-Einstein occupation number associated with photon energy

- Planck proposed the idea of light quanta to match the graph above for all frequency ranges.

  $$E_n = nh\nu$$

  Where n represents the quantum number for the photon where $n = 0,1,2,3 \ldots \infty$, and ν represents frequency. Average Energy $\bar{E}$ can be calculated by using the Botzman expression.

$$\bar{E} = \sum E_n P(E_n) = \frac{\sum_n E_n e^{-\frac{E_n}{k_B T}}}{\sum_n e^{-\frac{E_n}{k_B T}}} = \frac{\sum_n nh\nu\, e^{-\frac{nh\nu}{k_B T}}}{\sum_n e^{-\frac{nh\nu}{k_B T}}}$$

Using the following sums when $|x| < 1$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \; ; \; \sum_{n=0}^{\infty} nx^n = \frac{x}{(1-x)^2}$$

Since $0 < e^{-\frac{nh\nu}{k_B T}} < 1$ and consider $e^{-\frac{nh\nu}{k_B T}} = x$

$$\bar{E} = h\nu \frac{\sum_n nx^n}{\sum_n x^n} = h\nu \frac{\frac{x}{(1-x)^2}}{\frac{1}{1-x}} = h\nu \frac{x}{1-x} = h\nu \frac{1}{x^{-1}-1} = h\nu \frac{1}{e^{\frac{h\nu}{k_B T}}-1}$$

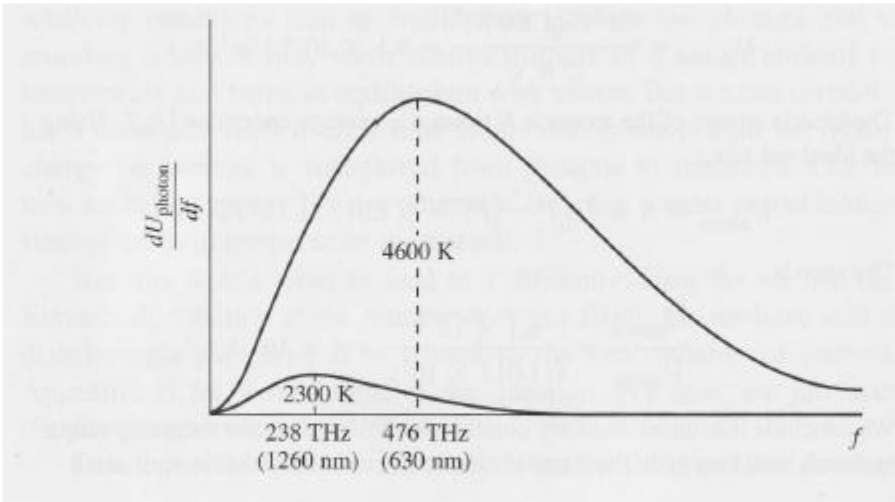
Now, take a look at the total energy of the photon system expression using the quantum statistics.

$$E = \int_0^{\infty} E\mathcal{N}(E)D(E)dE = \int_0^{\infty} \frac{E}{e^{E/k_B T} - 1}\left(\frac{8\pi V}{h^3 c^3} E^2\right) dE$$

Use $E \equiv k_B T x$

$$= \frac{8\pi V}{h^3c^3}\int_0^{\infty}(k_BT)^4\frac{x^3}{e^x-1}dx = \frac{8\pi V(k_BT)^4}{h^3c^3}\frac{\pi^4}{15} = \frac{8V\pi^5k_B^4}{h^3c^3}T^4$$

→ This is in agreement with Stephen Boltzmann's law



- Also, the above graph is based on the energy formula, we can see Wien's law results.

$$dE = \frac{h\nu^3}{e^{h\nu/k_BT}-1}\left(\frac{8\pi V}{c^3}\right)d\nu$$

(see 238 THz at 2300K → 476 THz at 4600K).

Compare Energy of Photons at the room temperature with Energy of a one Mole ideal gas:

- Energy of 1 mole ideal gas molecules at 273K, 1 atm
  $= \frac{3}{2}RT = \frac{3}{2}PV = \frac{3}{2}(1.013\ x\ 10^5 Pa)$V

- Energy of photon at 300K $= \int_0^{\infty} E\mathcal{N}(E)D(E)dE = \int_0^{\infty}\frac{E}{e^{k_BT}-1}\left(\frac{8\pi V}{h^3c^3}E^2\right)dE$

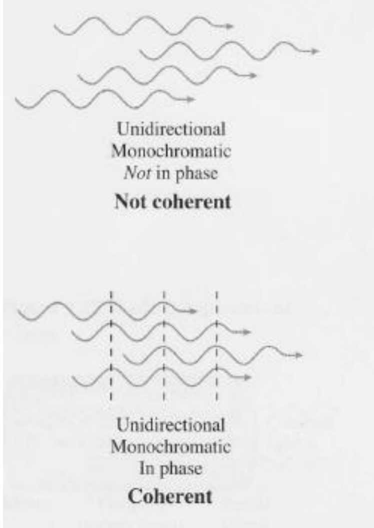
Use $E \equiv k_BTx$

$= \frac{8\pi V}{h^3c^3}\int_0^{\infty}(k_BT)^4\frac{x^3}{e^x-1}dx = \frac{8\pi V(k_BT)^4}{h^3c^3}\frac{\pi^4}{15} = 6.1\ x\ 10^{-6}\ J/m^3V$

→ is very small compared to energy of 1 mole ideal gas

Laser

Figure 9.21 Coherent versus incoherent light.

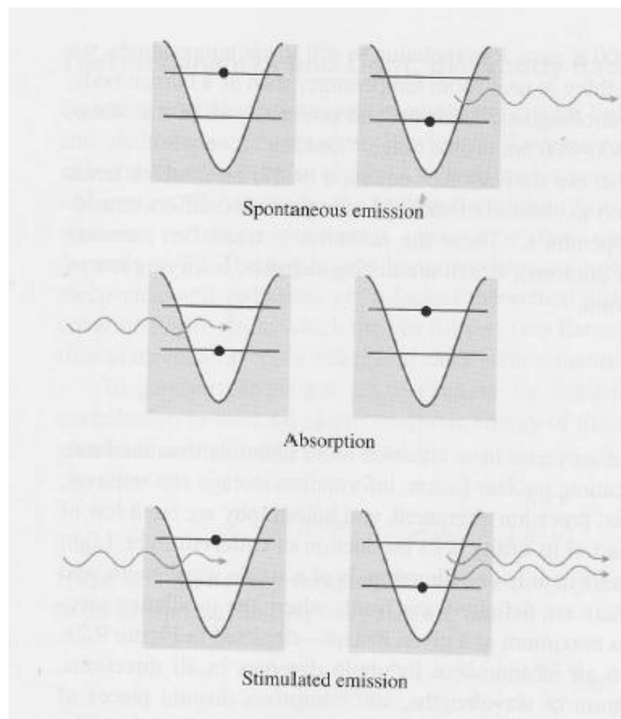


- LASER means "Light Amplification by the Stimulated Emission of Radiation"
- Light produced by the Laser is highly coherent. Light can be coherent when it travels in only one direction, is of a single wavelength, and is in phase.
- Spontaneous Emission: Light (Photon) is emitted from the higher energy level (E_2) to the lower energy level (E_1) with the photon energy, $h\nu$, which corresponds to the energy difference ($h\nu = E_2 - E_1$). The rate at which the spontaneous emission occurs from $E_2$to E_1 is proportional to how many particles are at E_2.

$$R_{spo} = A_{spo}N_2$$

- Absorption: Light (Photon) with the energy of $h\nu$ can be absorbed to promote the particles in the lower energy state (E_1) to the higher energy state (E_2). The rate at which the absorption to occur should be proportional to the number of particles at E_1 as well as the number of photons, $Y(\Delta E)$, having energy $h\nu$.

$$R_{abs} = B_{abs}N_1Y(\Delta E)$$



- Stimulated Emission: Similarly, the particles at E_2 can emit photons with $h\nu$ when stimulated by the presence of $h\nu$. The rate at which the stimulated emission occurs is proportional to N_2 as well as as well as the number of photons, $Y(\Delta E)$, having energy $h\nu$.

$$R_{sti} = B_{sti}N_2Y(\Delta E)$$

- Principle of detailed balance:
  The emission rate = The absorption rate

$$R_{spo} + R_{sti} = R_{abs}$$

$$A_{spo}N_2 + B_{sti}N_2Y(\Delta E) = B_{abs}N_1Y(\Delta E)$$

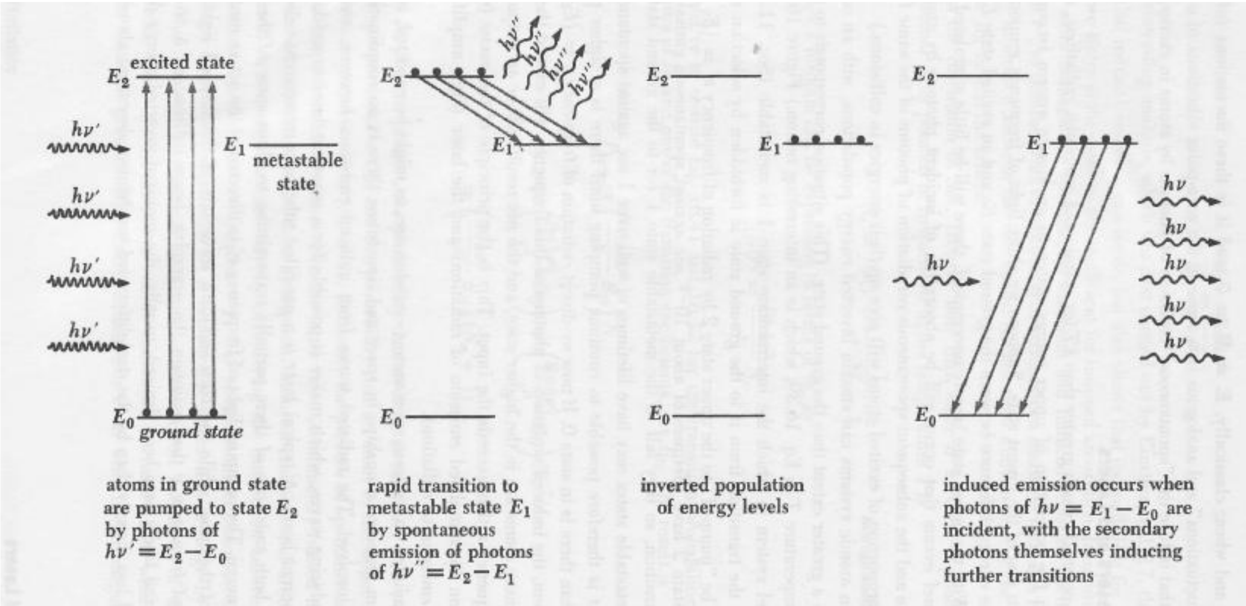
$$(B_{abs}N_1 - B_{sti}N_2Y)Y(\Delta E) = A_{spo}N_2$$

$$Y(\Delta E) = \frac{A_{spo}N_2}{(B_{abs}N_1 - B_{sti}N_2Y)} = \frac{A_{spo}/B_{abs}}{\frac{N_1}{N_2} - \frac{B_{sti}}{B_{abs}}} \quad = \quad \frac{A_{spo}/B_{abs}}{e^{\Delta E/k_BT} - \frac{B_{sti}}{B_{abs}}}$$

$$\text{Since } N_1 \propto e^{-\frac{E_1}{k_BT}} \text{ and } N_2 \propto e^{-\frac{E_2}{k_BT}}$$

This formula represents the number of photons with the energy difference of $h\nu$, knowing photon is a boson, then $B_{sti} = B_{abs}$. We can have photon energy of $h\nu$ that can be equally likely to be absorbed by the particles in the lower energy state and to cause the stimulated emission of the particles in the higher energy state.

- Population Inversion: Increase the number of particles at the higher energy state. We cannot simply increase the number of particles at the higher energy state because the occupation number drops exponentially as energy increases.
- Use of a metastable state, E_1.
    - Optical pumping with photons with energy $h\nu'(= E_2 - E_0)$leads the equilibrium state where $N_0 = N_2$
    - The particles at E_2has a preferential drop to the E_1state than the E_0state, emitting $h\nu'' = E_2 - E_1$, making $N_1 > N_0$.
    - When photons with $h\nu = E_1 - E_0$ are present, they can induce stimulated emission of coherent photons with $h\nu$.



- Laser examples:
    - Ruby lasers: three level with optical pumping produced 693.4 nm light.
    - Four level neodymium glass lasers produce 1060 nm.