PH102: Modern Physics Homework 1 (Due: 1/20/2012)

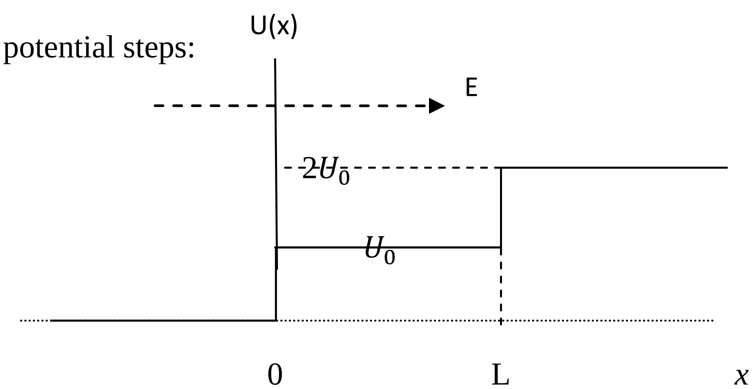
1. **(10 points)** Consider a free wave function $\psi(x) = A\exp(ikx)$ with k being a real number.
   (a) Show that this wave function is not normalizable.
   (b) Discuss how this problem can be fixed by considering
      (i) (Required) the fact that the space is not really infinite

      (ii) (Extra credit) the fact that in a real experiment a perfectly monochromatic wave can never be prepared.

      Note. Discussion of the fact (i) must be quantitative, but discussion of the fact (ii) can be qualitative (you might need some research on "wave packet").

2. **(20 points)** Consider the following potential steps:

$$U(x) = \begin{cases} 0 & x < 0 \\ U_0 & 0 \le x < L \\ 2U_0 & x \ge \mathrm{L} \end{cases}$$



   (a) Write the time-independent form of the Schrodinger Equation in three regions: $x < 0$, $0 \le x < L, and\ x \ge \mathrm{L}$.
   (b) The electron is impinging from the left with energy $E > 2\,U_0$. Write down the wave function $\psi(x)$ in the three regions.
   (c) What conditions should $\psi(x)$ meet at the boundaries, $= 0\ and\ x = L$? What do these boundary conditions tell about the relationships among coefficients of wave function components in $\psi(x)$?
   (d) Define R (reflection probability) and T (transmission probability) using the coefficients of wave function components.

3. **(10 points)** A particle in a 2 dimensional infinite well
   (a) Write the time-independent Schrodinger Equation for a particle in the two dimensional infinite well. The dimension of the box is a long and b wide.
   (b) Solve the Schrodinger Equation and obtain allowed energy values and associated wave functions.
   (c) Consider the special case $a = b$. Find one energy value that is non-degenerate and another energy value that is degenerate. For the latter, find the degeneracy.

4. **(10 points)** Consider the free electron wave in the presence of a single potential step.

$$U\ (x) = 0 \qquad x < 0$$
$$U(x) = U_0 \qquad x \geq 0$$

The initial beam is coming from the left and the wave function is given by

$$\psi(x) = \exp(ikx) + B\exp(-ikx) \qquad x < 0$$
$$\psi(x) = C\exp(ik'x) \qquad x \geq 0$$

Assume that $E > U_0$.

(a) Show that $B > 0$ when $U_0 < 0$ but $B < 0$ when $U_0 > 0$. [Hint: page 6 of Lecture note 2; you do need to solve for B in terms of k, k'.]

(b) Discuss the similarity of this situation with a well-known fact that when light propagates in medium with refractive index n_1 and reflects off of another medium with reflective index n_2 (at normal incidence), the phase changes by π when $n_2 > n_1$, but the phase does not change when $n_1 > n_2$. In your discussion, you may find it useful to correlate the phase change with the slowing down or the speeding up of the particle (electron or photon) as it enters the new medium.

5. **(10 points)** Consider the Resonant Transmission Condition for the potential barrier problem. (Page 9 of Lecture note 2). Using the result from the previous problem and analyzing the problem like an optics problem, explain why the transmission becomes 1 and the reflection becomes 0 when the resonance condition $k'L = n\,\pi$ is met. Here is what I mean by treating the problem like a optics problem: you can consider a transmitted beam of electrons as the superposition of (1) the initial beam that just propagates without any reflection at the potential boundaries (so this is a completely undisturbed beam), (2) the initial beam that reflects off of the right boundary ($x = L$) and then off the left boundary ($x = 0$) to emerge to the right region (define these two successive reflections as one "bounce") (3) the initial beam that emerges to the right region after two bounces, (4) the initial beam that emerges to the right region after three bounces, etc. etc. Consider the constructive interference condition for these waves. Apply a similar analytic reasoning for the reflected beam ($B\exp(-ikx)$). [This way of analysis happens to be very close to the celebrated Feynman view of quantum mechanics!]