PH102, 2012W, Lecture Notes: March 7, Wed, Class 21

Radioactive decay:

- Definition: An unstable nucleus (parent nucleus) can spontaneously emit small particles or energies to become a nucleus (daughter nucleus) in a more stable state.
- Energy is conserved in radioactive decay: $Q\ (kinetic\ energy\ released) = (m_i - m_f)c^2$

Three forms of radioactive decay depending upon what is emitted during radioactive decay

Alpha decay

- Emits an alpha particle (He nucleus=2 protons + 2 neutrons)
- This process makes:

$$^{A}_{Z}X(parent) \rightarrow {}^{A-4}_{Z-2}Y(daughter) + {}^{4}_{2}He$$

  - $Z_{daughter} = Z_{parent} - 2$
  - $N_{daughter} = N_{parent} - 2$
  - $A_{daughter} = A_{parent} - 4$

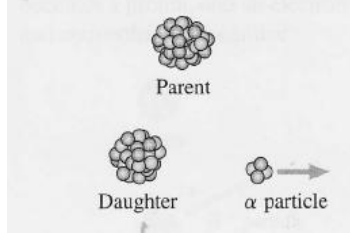


Figure 11.18 Alpha decay.

- Released kinetic energy (Q)

$$Q = (m_{parent} - m_{daughter} - m_{^4_2He})\ c^2$$

- Example: alpha decay of $^{238}_{92}U$

$$^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$$
$$Q = (m_{parent} - m_{daughter} - m_{^4_2He})\ c^2$$
$$= (238.050784 - 234.043593 - 4.002603)uc^2$$
$$= 0.004588 \text{ x } 931.5\ MeV = 4.27\ MeV$$

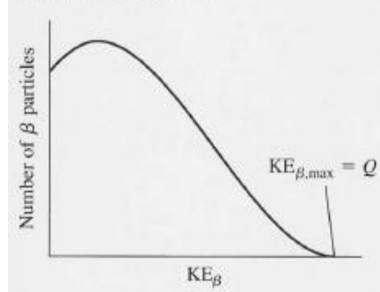
Since the mass of He is a lot smaller than that of Th, the alpha particle gets almost all the kinetic energy during the alpha decay process.

Beta decay emits temporarily created negatively charged electrons (β^- *particle*) or positively-charged electrons (β^+ *particle*).

In order to satisfy charge conservation, energy conservation, and angular momentum conservation, a new particle is introduced called neutrinos/antineutrinos.

- Charge conservation: since beta particles carry a charge, the charge of the parent nucleus should increase by 1 in β^- decay and decrease by 1 in β^+ decay. As a result, neutrinos' charge should be neutral.
- Angular momentum conservation: since β particles have a spin of ½, to conserve angular momentum, neutrinos should have a spin of ½.
- Energy conservation. Figure 11.19 shows that kinetic energies of β particles emitted during the decay vary greatly from zero to Q (maximum allowed). Therefore, neutrinos should carry portions of kinetic energies with β particles. To allow β particles to have the maximum allowed kinetic energy, neutrinos' mass should be negligible.

Figure 11.19 The mysterious variation in β particle energies.



<table>
<tr><th>$\boldsymbol{\beta^-}$ decay</th><th>$\boldsymbol{\beta^+}$ decay</th></tr>
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- An electron ($\boldsymbol{\beta^-}$ $\boldsymbol{particle}$) is temporarily created and then emitted.
- Emits an electron and an anti-neutrino.
- Changes a neutron inside the nucleus into a proton.

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- Emits a positron and a neutrino
- Changes a proton inside the nucleus into a neutron.

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$${}^{A}_{Z}X(parent) \rightarrow {}_{Z+1}^{\;\;A}Y(daughter) + {}_{-\mathbf{1}}^{\;\;\mathbf{0}}\boldsymbol{\beta}^- + \bar{\boldsymbol{\nu}}$$

- $Z_{daughter} = Z_{parent} + 1$
- $N_{daughter} = N_{parent} - 1$
- $A_{daughter} = A_{parent}$

Released kinetic energy

$$Q = (m_{parent} - m_{daughter})\, c^2$$

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$${}^{A}_{Z}X(parent) \rightarrow {}_{Z-1}^{\;\;A}Y(daughter) + {}_{+\mathbf{1}}^{\;\;\mathbf{0}}\boldsymbol{\beta}^+ + \boldsymbol{\nu}$$

- $Z_{daughter} = Z_{parent} - 1$
- $N_{daughter} = N_{parent} + 1$
- $A_{daughter} = A_{parent}$

Released kinetic energy

$$Q = (m_{parent} - m_{daughter} - 2m_{electron})\, c^2$$

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Example:

$${}^{12}_{5}B \rightarrow {}^{12}_{6}C + {}_{-\mathbf{1}}^{\;\;\mathbf{0}}\boldsymbol{\beta}^- + \bar{\boldsymbol{\nu}}$$

$$Q = (12.014352 - 12)uc^2 = 13.4\ MeV$$

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$${}^{12}_{7}N \rightarrow {}^{12}_{6}C + {}_{+\mathbf{1}}^{\;\;\mathbf{0}}\boldsymbol{\beta}^+ + \boldsymbol{\nu}$$

$$Q = (12.018613 - 12 - 2 \text{ x } 0.0005486)uc^2 = 16.3\ MeV$$

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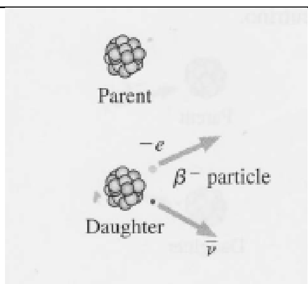

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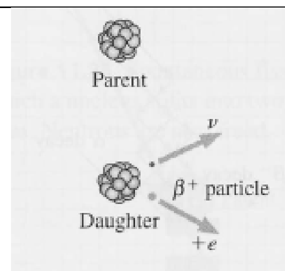

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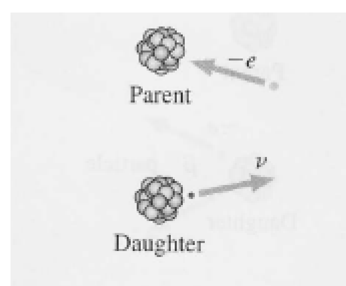
The third form of beta decay is Electron Capture.

- A nucleus with too many protons can change a proton into a neutron by capturing an electron.
- Electron capture is easier than $\boldsymbol{\beta^+}$decay since an electron is already exists for a nucleus to capture.



$${}^{A}_{Z}X(parent) + {}_{-\mathbf{1}}^{\;\;\mathbf{0}}\boldsymbol{\beta}^- \rightarrow {}_{Z-1}^{\;\;A}Y(daughter) + \nu$$

- $Z_{daughter} = Z_{parent} - 1$
- $N_{daughter} = N_{parent} + 1$
- $A_{daughter} = A_{parent}$

- Released kinetic energy

$$Q = (m_{parent} - m_{daughter})\, c^2$$

- Example:

$${}^{11}_{6}C + {}_{-\mathbf{1}}^{\;\;\mathbf{0}}\boldsymbol{\beta}^- \rightarrow {}^{11}_{5}B + \nu$$

$$Q = (11.01143 - 11.009305)uc^2 = 1.97\ MeV$$

Gamma Decay

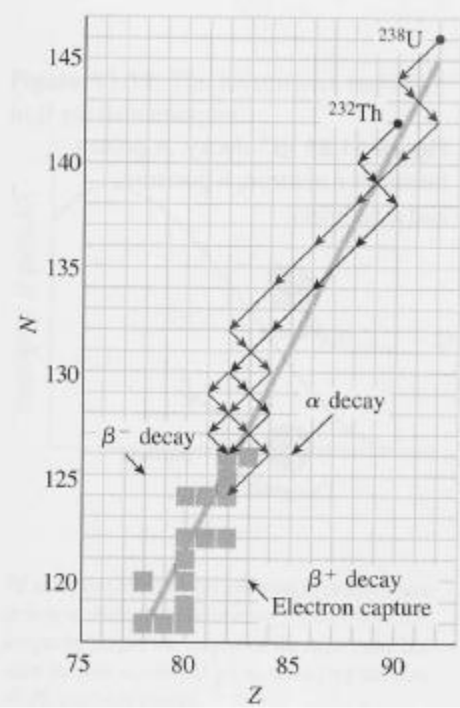
- A nucleus in an excited state emits photons (gamma particles, γ) to go into a lower energy state.
- Gamma decay does not alter N or Z.
- Gamma energies are characteristic of a given isotope, and are thus used to identify the isotope.

Figure 11.24 Gamma decay.

Decay series

- An unstable nucleus can be involved in a series of decays until it finds a stable state.
- We can plot this process on a graph that represents N and Z numbers of each nucleus in the series.
- Figure 11.23 shows such a graph. The gray line represents the line of stability. The figure shows two series: one for U-238 and the other for Th232.
- Alpha decay is shown by an arrow
    - $Z_{daughter} = Z_{parent} - 2$
    - $N_{daughter} = N_{parent} - 2$
- $\boldsymbol{\beta^-}$ **decay**
    - $Z_{daughter} = Z_{parent} + 1$
    - $N_{daughter} = N_{parent} - 1$
- $\boldsymbol{\beta^+}$ **decay and electron capture**
    - $Z_{daughter} = Z_{parent} - 1$
    - $N_{daughter} = N_{parent} + 1$

Figure 11.23 The "directions" of α and β decays, and the decay series of uranium-238 and thorium-232.



Radioactive Decay Law

- For all decays, the rate of decay over time will be proportional to the sample size:

$$\frac{dN}{dt} \propto N$$

$$\frac{dN}{dt} = -\lambda N$$ where N=Number of nuclei; λ=decay constant

$$\frac{dN}{N} = -\lambda\, dt$$

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt$$

$$ln\frac{N}{N_0} = -\lambda t$$

$$N = N_0 e^{-\lambda t}$$

- Decay rate R = λN (decays per second)
- A sample of the same nuclei will decay by the same fraction in equal successive intervals of time.
- Half-life ($T_{1/2}$) is defined as the time interval at which half of the sample will decay:

$$\frac{1}{2}N_0 = N_0 e^{-\lambda T_{1/2}}$$

From this relationship, we can calculate decay constant

$$\lambda = \frac{ln2}{T_{1/2}}$$

Figure 11.26 shows the relationship between half life and the number of nuclei remaining over time.

- Half-lives vary widely, from 10^{-22} seconds to 10^{+17} years.

Figure 11.26 Exponential decay.

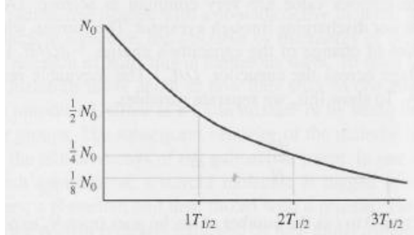


TABLE 11.3 Selected decays

Isotope	Decay Mode	Half-Life
$^{35}_{20}Ca$	β^+	50 ms
$^{3}_{1}H$	β^-	12.3 yr
$^{238}_{92}U$	α	4.5×10^9 yr

Example:
A sample holds 2 μg of tritium.

- Initial decay rate (R)

$$R = \lambda N = \frac{\ln 2}{T_{1/2}} \cdot \frac{sample\ mass}{atomic\ mass\ of\ Tritium}$$

Since Tritium's atomic mass = 3.02 u and 1 u=1.66 x $10^{-27} kg$, $T_{1/2} = 12.3\ yr = 12.3 \text{ x } 3.16 \text{ x } 10^7 sec$

$$\lambda = \frac{\ln 2}{T_{1/2}} = 1.78\ x\ 10^{-9} \quad \text{/sec}$$

$$R = 7.1 \text{ x } 10^8\ decays/sec$$

- Elapsed time before the decay rate falls to 1% of its initial value?

$$N = N_0 e^{-\lambda t}$$

$$\frac{1}{100} N_0 = N_0 e^{-\lambda t}$$

$$t = \frac{-\ln\left(\frac{1}{100}\right)}{\lambda} = 2.6\ x\ 10^9 \text{ sec} = 81.7\ years$$

Carbon-14 dating

- Carbon-14's β^-decay has a half-life of 5730 years.
- Carbon-14 dating only works for formerly living organisms.
- Carbon-14's amount is constantly maintained for living organisms since living organisms exchange Carbon with the environment. Ratio of naturally produced C14/C12=1.3 x 10^{-12}
- When, a living organism dies, it stops the exchange process, thus C-14 in the dead organism decays exponentially.

Example:
What is the age of a fossil sample that contains 6 g of carbon and has a decay rate (R) of 30 decays per minute?

- The sample has 6 g of Carbon. 1.3 x 10^{-12} th of 6g of Carbon should be C-14 at the time when the organism in the sample was alive. This amount of C-14 is subject to the decay. Therefore,

$$N_0(C_{14}) = (1.3x\ 10^{-12}) \cdot \left(\frac{1}{2}\right) \cdot (6.02 \text{ x } 10^{23}) = 3.9 \text{ x } 10^{11}$$

- Decay constant

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{5730\ years \cdot 3.16\ x10^{7} sec/year} = 3.83 \text{ x } 10^{-12}\ \ /sec$$

- Current decay rate (R)=30 decays/60 seconds=1/2 decays/sec

$$R = N \cdot \lambda = N \cdot 3.83 \text{ x } 10^{-12}\ \ /sec$$

$$N = 1.31 \text{ x } 10^{11}$$

$$N = N_0 e^{-\lambda t}$$

$$t = -\frac{1}{\lambda} \ln \frac{N}{N_0} = -\frac{1}{3.83 \text{ x } 10^{-12}\ \ /sec} \ln \frac{1.31 \text{ x } 10^{11}}{3.9 \text{ x } 10^{11}} = 2.86 \text{ x } 10^{11}\ sec \sim 9000\ years$$