PH102, 2012W, Lecture Notes: March 5, Mon, Class 20

Nuclear Physics: Nuclear Shell Model

The Liquid Drop model can predict the nuclear binding energy reasonably well. The Liquid Drop model is based on the Strong Force between two adjacent nucleons in a nucleus. However, the Liquid Drop model cannot explain magic numbers: 2, 8, 20, 28, 50, 82, 126 where nuclei with magic numbers of nucleons are particularly stable. The Shell Model explains the magic numbers, the tendency for Z and N to be simply even numbers, and the model can explain angular momenta of nuclei.

	Atomic Shell Model	Nuclear Shell Model
Potential	Electrostatic between nucleus and electrons	Net effect of all the forces nucleons experience in a nucleus
Magic numbers	Atoms with closed electronic shells are stable such as He (2 electrons), Ne (10), Ar (18), Kr (36), Xe (54), Rn (86).	Nuclei are particularly stable when the number of nucleons is 2, 8, 20, 28, 50, 82, and 126.
Exclusion principle	Electrons follow the Exclusion principle	Protons and neutrons separately follow the Exclusion Principle
Movement	Electrons move in orbitals	Nucleons do not move like electrons because most nucleons fill states to a maximum level, preventing them from changing momentum.

Atomic Energy States:

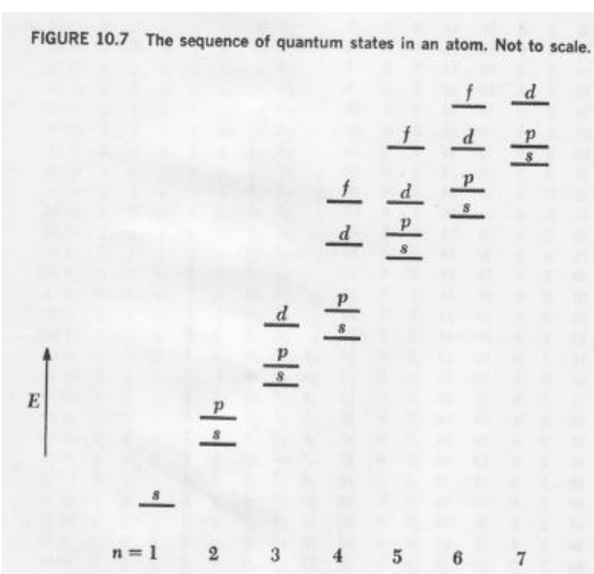
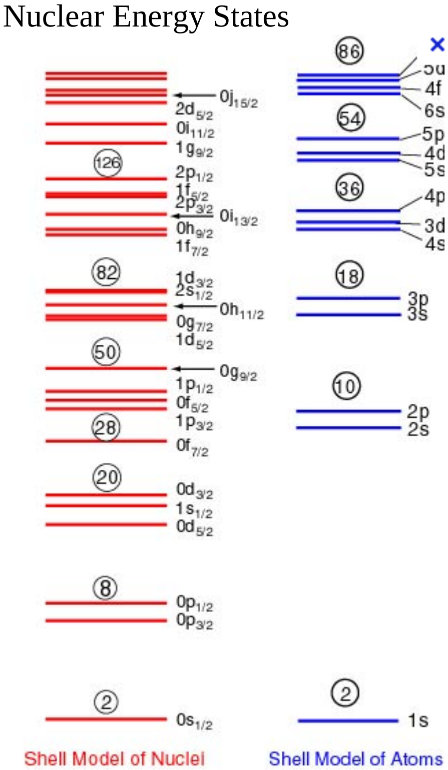


FIGURE 10.7 The sequence of quantum states in an atom. Not to scale.

Nuclear Energy States



The nuclear shell model is based on three dimensional harmonic oscillator solutions.

$$\frac{-\hbar^2}{2m}\nabla^2\psi(x)+\frac{1}{2}\mu\omega^2x^2\,\psi(x) = \mathrm{E}\,\psi(x)$$

$$\psi_{klm}(r,\theta,\phi)=N_{kl}r^{l}e^{-\nu r^2}L_k^{(l+\frac{1}{2})}(2\nu r^2)Y_{lm}(\theta,\phi)$$

where

$$N_{kl}=\sqrt{\sqrt{\frac{2\nu^3}{\pi}}\frac{2^{k+2l+3}\,k!\,\nu^l}{(2k+2l+1)!!}}$$ is a normalization constant.

$$\nu\equiv\frac{\mu\omega}{2\hbar}$$

$L_k^{(l+\frac{1}{2})}(2\nu r^2)$ are generalized Laguerre polynomials. The order k of the polynomial is a non-negative integer.

$Y_{lm}(\theta,\phi)$ is a spherical harmonic function.

The energy eigenvalue is

$$E=\hbar\omega\left(2k+l+\frac{3}{2}\right)$$

The energy is usually described by the single quantum number

$$n\equiv 2k+l$$

- For every even $n, l = 0, 2, \ldots n-2, n$
- For every odd $n, l = 1, 3, \ldots n-2, n$
- $-l \le m \le l$
- Every n and l, there are $2l+1$ energy degeneracies, which can accommodate $2(2l+1)$ nucleons

Using these rules, we can obtain the following table:

n	k	l	No. of nucleons in (n, k, l)	No. of nucleons in n	Total nucleons	Energy
0	0	0	2	2	2	$\frac{3}{2}\hbar\omega$
1	0	1	6	6	8	$\frac{5}{2}\hbar\omega$
2	0	2	10	12	20	$\frac{7}{2}\hbar\omega$
	1	0	2			
3	0	3	14	20	40	$\frac{9}{2}\hbar\omega$
	1	1	6			
4	0	4	18	30	70	$\frac{11}{2}\hbar\omega$
	1	2	10			
	2	0	2			
5	0	5	22	42	112	$\frac{13}{2}\hbar\omega$
	1	3	14			
	2	1	6			

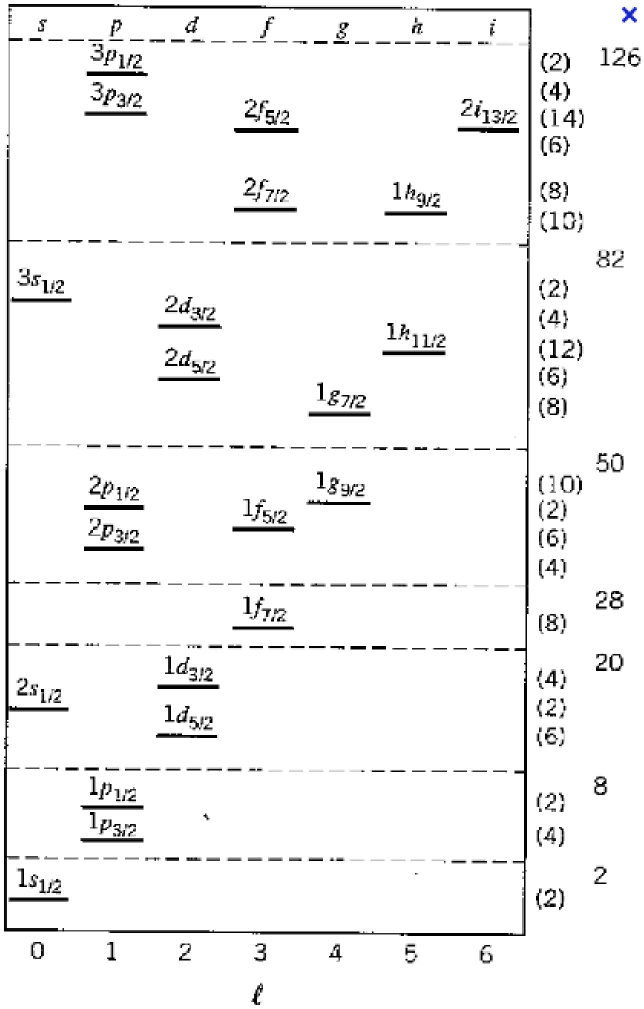
The above table predicts the magic numbers of 2, 8, and 20, but not higher than these numbers. To account for this, we need to consider the L-S coupling, which is similar to the L-S coupling of electrons.

$$\vec{J} = \vec{L} + \vec{S}$$

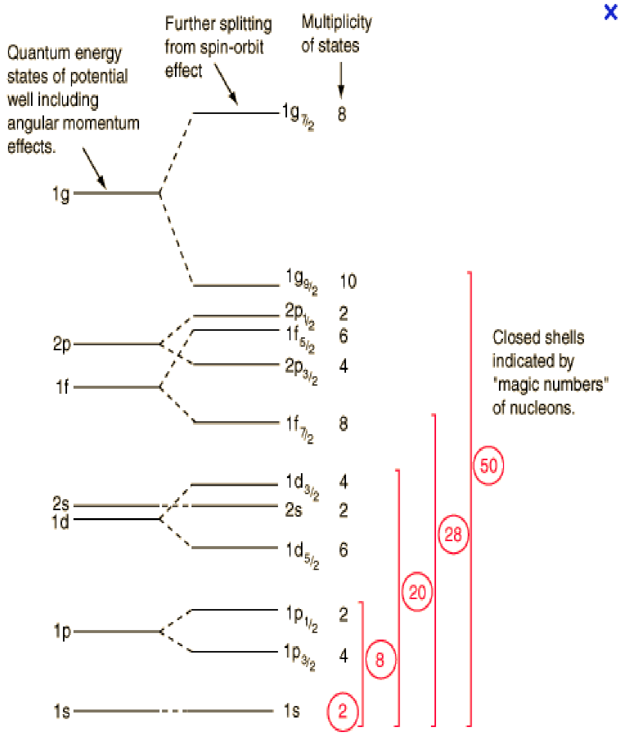
This means that, instead of l and m_l and spin (as two times of each state), we need to consider j and m_j. This will result in each (n,k,l) energy state to split into two more states.

n	k	l	j	No. of nucleons in (n, j)	No. of nucleons in (n, j)	Energy
0	0	0	½	2	2	$\frac{3}{2}\hbar\omega$
1	0	1	3/2 1/2	4 2	6	$\frac{5}{2}\hbar\omega$
2	0	2	5/2 3/2	6 4	10	$\frac{7}{2}\hbar\omega$
	1	0	½	2	2	
3	0	3	7/2 5/2	8 6	14	$\frac{9}{2}\hbar\omega$
	1	1	3/2 1/2	4 2	6	
4	0	4	9/2 7/2	10 8	18	$\frac{11}{2}\hbar\omega$
	1	2	5/2 3/2	6 4	10	
	2	0	½	2	2	
5	0	5	11/2 9/2	12 10	22	$\frac{13}{2}\hbar\omega$
	1	3	7/2 5/2	8 6	14	
	2	1	3/2 1/2	4 2	6	

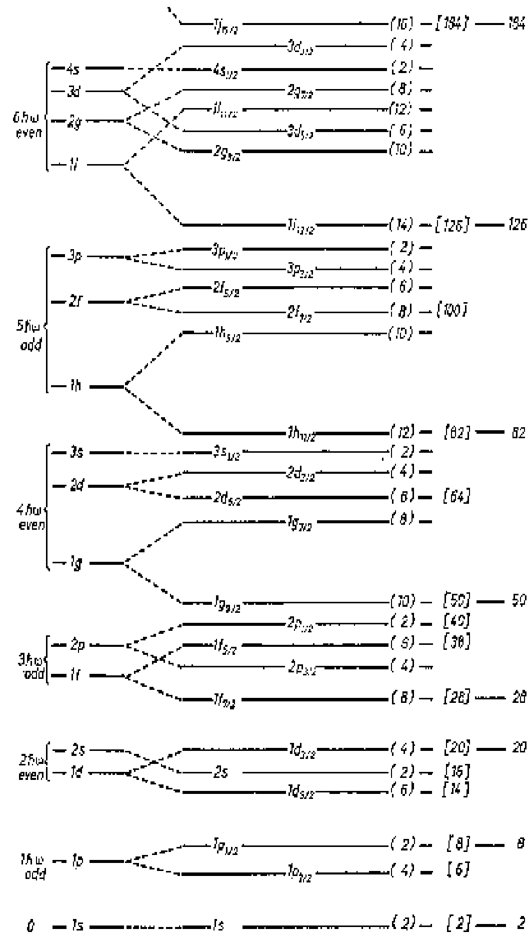
Then, each (n, k, l) state splits into two j states, except $j = 0$. When it does, the split becomes greater as l gets greater for the same k. In each split, the higher j state is lower than the lower j state.



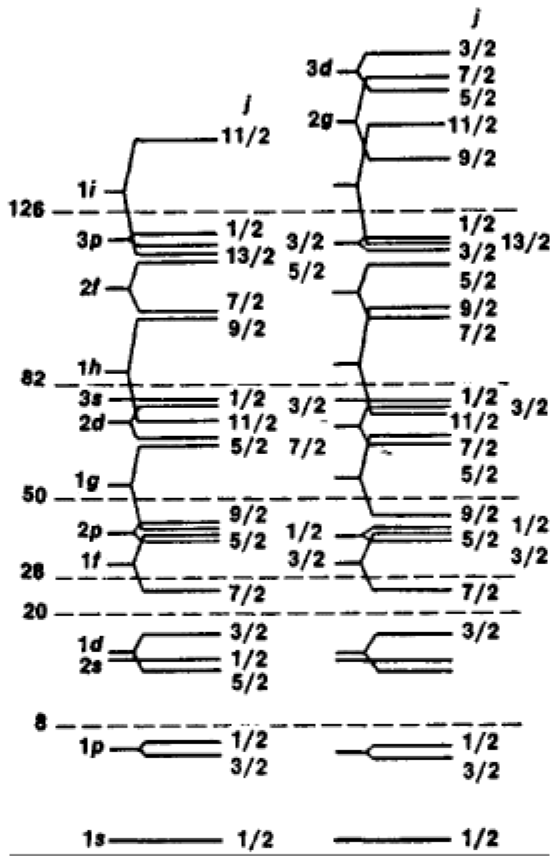
s p d f g h i
3p1/2
3p3/2
2f5/2
2i13/2
2f7/2
1h9/2
3s1/2
2d3/2
1h11/2
2d5/2
1g7/2
2p1/2
1g9/2
1f5/2
2p3/2
1f7/2
1d3/2
2s1/2
1d5/2
1p1/2
1p3/2
1s1/2
(2) (4) (14) (6) (8) (10)
(2) (4) (12) (6) (8)
(10) (2) (6) (4)
(8)
(4) (2) (6)
(2) (4)
(2)
126
82
50
28
20
8
2
0 1 2 3 4 5 6
ℓ



Quantum energy states of potential well including angular momentum effects.
Further splitting from spin-orbit effect
Multiplicity of states
1g
2p
1f
2s
1d
1p
1s
1g7/2 8
1g9/2 10
2p1/2 2
1f5/2 6
2p3/2 4
1f7/2 8
1d3/2 4
2s 2
1d5/2 6
1p1/2 2
1p3/2 4
1s
2
8
20
28
50
Closed shells indicated by "magic numbers" of nucleons.



184
126
82
50
28
20
8
2



Empirical evidence

Proton energy levels Neutron energy levels