Lecture 18

Superconductivity

Superconductivity

- Perfect diamagnetism
- Perfect conductivity

What we will learn

- Diamagnetic character
- Microscopic origin (Cooper pair, BCS)
- <u>Macroscopically coherent</u> quantum state (BEC)
- Current density and wave function
- Different types of superconductors
- Flux quantization

Diamagnetism??

Table of 'macroscopic' equations

Formulation in terms of *free* charge and current

Name	Differential form	Integral form
Gauss's law	$\nabla \cdot \mathbf{D} = \rho_f$	$\oiint_{\partial V} \mathbf{D} \cdot \mathrm{d}\mathbf{A} = Q_f(V)$
Gauss's law for magnetism	$\nabla \cdot \mathbf{B} = 0$	$\oiint_{\partial V} \mathbf{B} \cdot \mathrm{d}\mathbf{A} = 0$
Maxwell–Faraday equation (Faraday's law of induction)	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint_{\partial S} \mathbf{E} \cdot \mathrm{d}\mathbf{l} = -\frac{\partial \Phi_S(\mathbf{B})}{\partial t}$
Ampère's circuital law (with Maxwell's correction)	$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$	$\oint_{\partial S} \mathbf{H} \cdot \mathrm{d}\mathbf{l} = I_{f,S} + \frac{\partial \Phi_S(\mathbf{D})}{\partial t}$

$$\rho_b = -\nabla \cdot \mathbf{P},$$
$$\mathbf{J}_b = \nabla \times \mathbf{M} + \frac{\partial \mathbf{P}}{\partial t},$$
$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P},$$
$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}),$$
$$\rho = \rho_b + \rho_f,$$
$$\mathbf{J} = \mathbf{J}_b + \mathbf{J}_f,$$

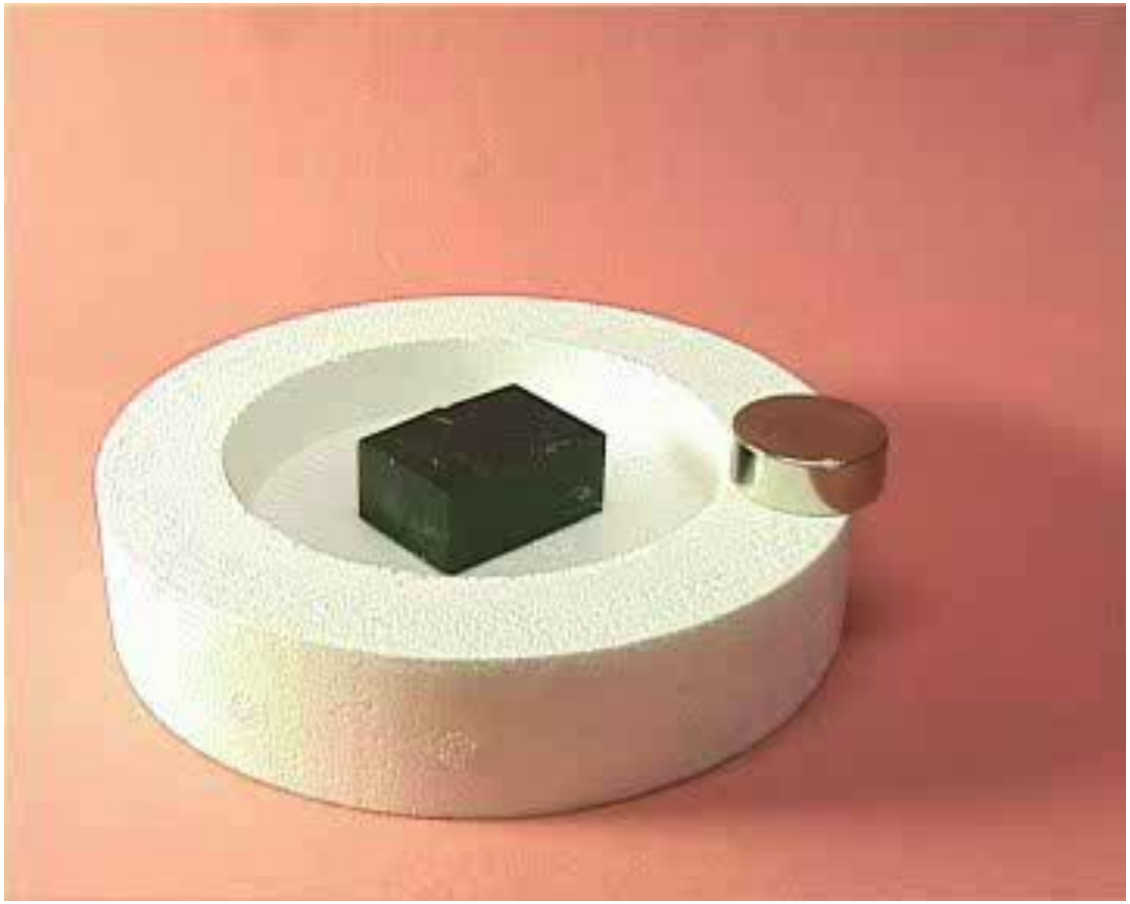
$\vec{M}$ and $\vec{H}$

Anti-parallel!

where **P** and **M** are polarization and magnetization, and ρ_b and $\mathbf{J}_b$ are bound charge and current, respectively.

Meissner Effect (perfect diamagnet)

$\vec{M} = -\vec{H} \Rightarrow \vec{B} = 0$



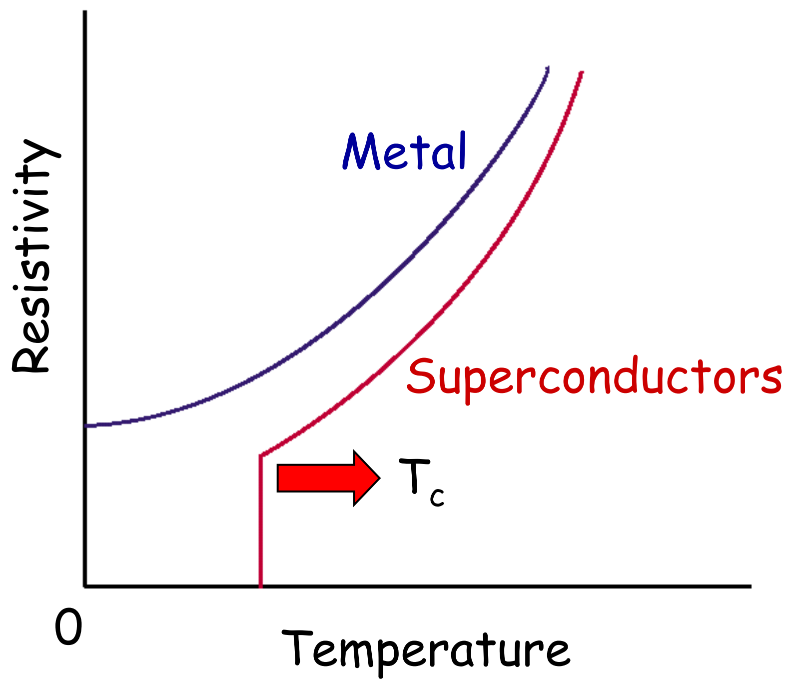
http://www.fys.uio.no/super/levitation/

Essential Characteristics

1911 **K. Onnes Superconductivity in Hg**

1933 **Meissner effect**

RESISTANCELESS CONDUCTION



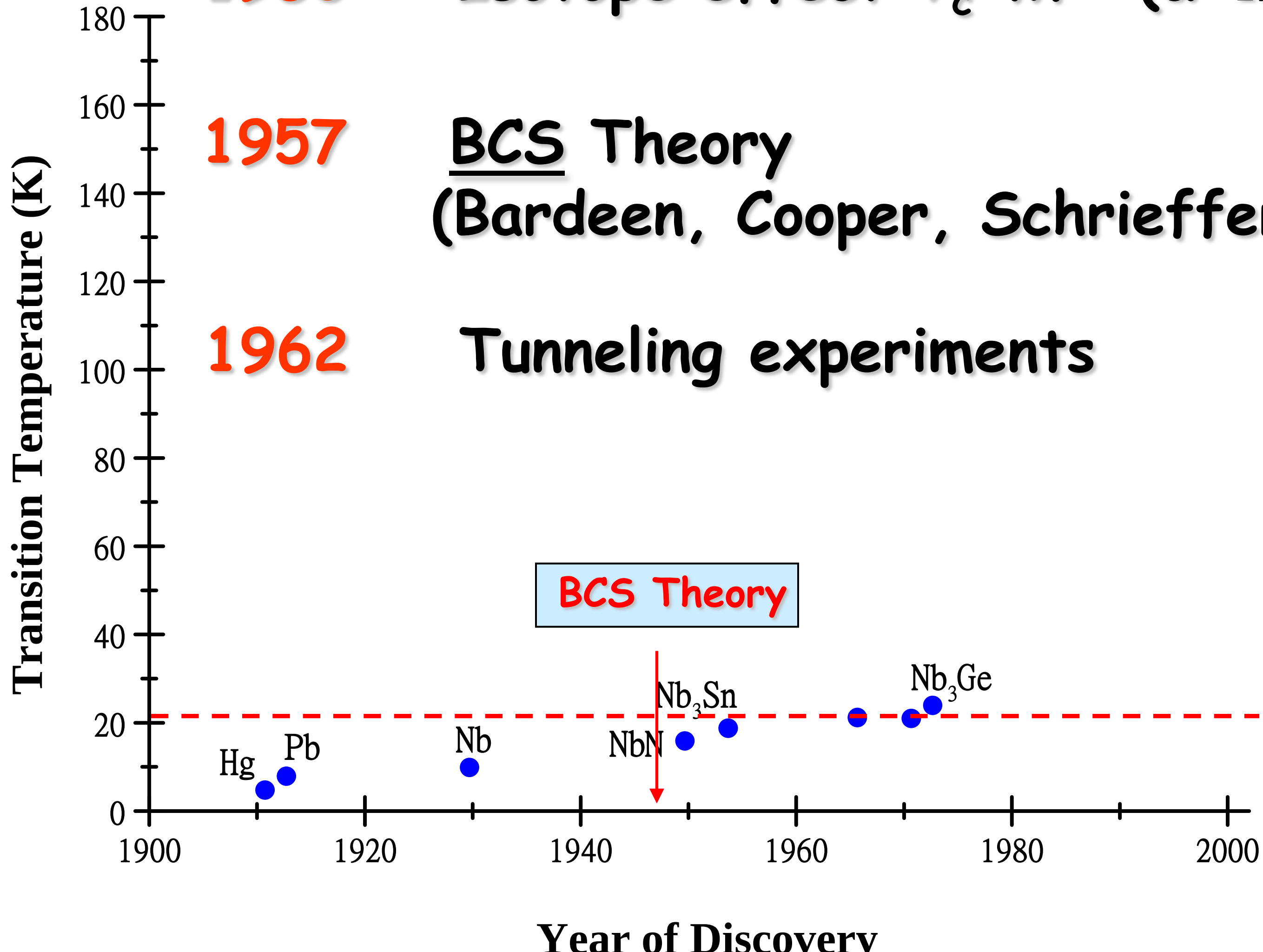
MEISSNER EFFECT:
Perfect diamagnetism

Understanding of Superconductivity

1950 Isotope effect $T_c \sim M^{-\alpha}$ ($\alpha \sim 1/2$)

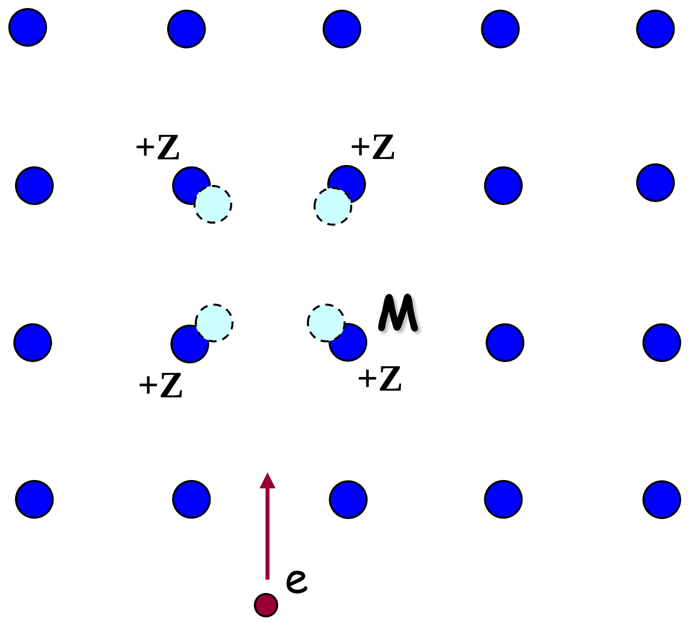
1957 <u>BCS</u> Theory (Bardeen, Cooper, Schrieffer)

1962 Tunneling experiments



BCS theory

PHONON MEDIATED PAIRING
(phonon = lattice vibration)



Pairs of electrons: Cooper pairs

Superconducting gap: Δ

El-ph coupling constant: λ

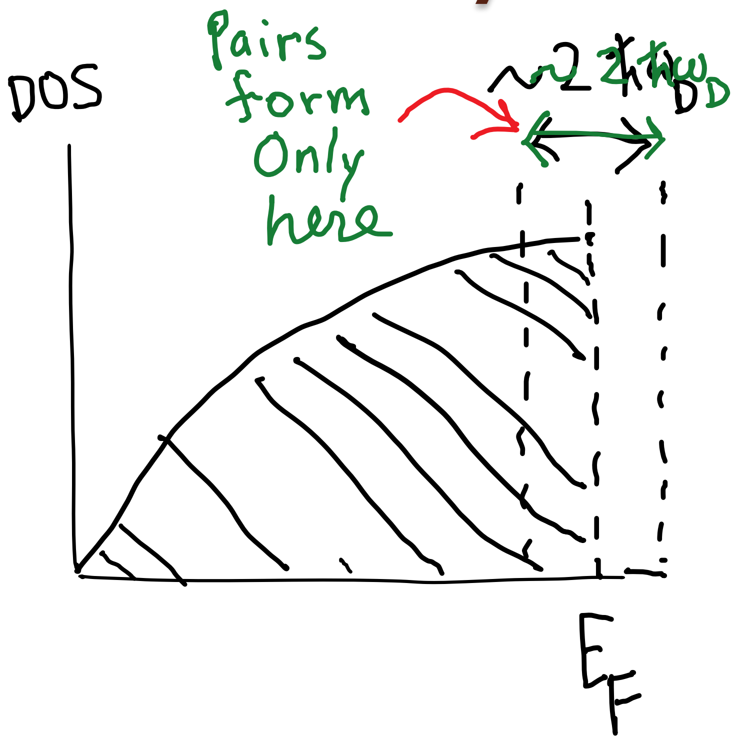
$$T_c \sim \omega_{ph} \exp(-1/\lambda) \sim M^{-1/2}$$

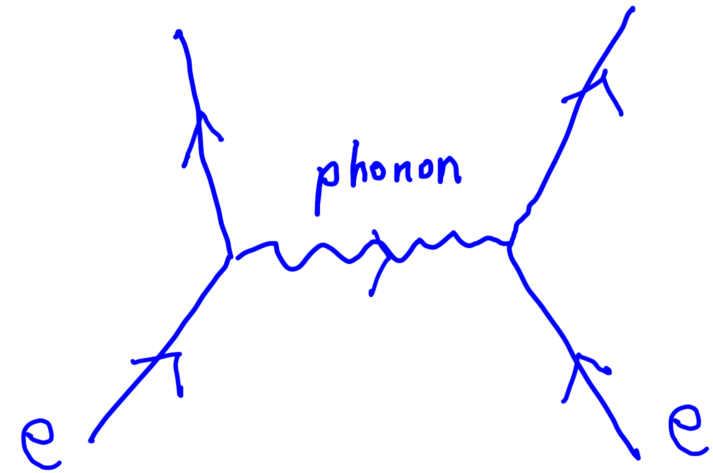
e-ph wins e-e at low freq. (Slow Wins!)

How many electrons participate?

- Not all electrons can participate in Cooper pairing.
- In quantum mechanical view, the interaction described in the previous slide is described as two electrons exchanging a phonon (lattice vibration quantum).

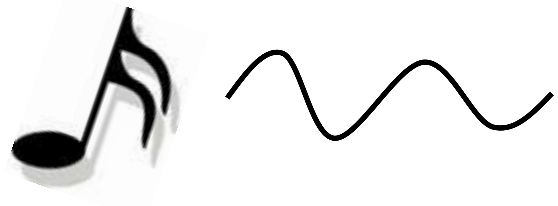
How many electrons participate?





Only those electrons near E_F form Cooper pairs. These are the electrons with energies $E = E_F \pm \hbar\omega_D$ where ω_D is the Debye frequency (typical frequency for phonons). Why? Due to Pauli exclusion principle, electrons cannot scatter by absorbing or emitting a phonon when they are deep inside the Fermi sea ($E \ll E_F$). So, only a fraction, $O\left(\frac{\hbar\omega_D}{E_F}\right)$, of total electrons participate in Cooper pairs.

Superconductivity – Dance of Electron Pairs



Origin of SC = **PAIRS** dancing to the **SAME** tune

Phys. Today, March '04

Superconducting Order Parameter (Ginzburg-Landau; Gorkov)

- Can be viewed as the "wave function" of the condensate (i.e. <u>ALL</u> electron pairs)
- All electron pairs (**<u>Cooper pairs</u>**) have <u>the same wave function</u>, in particular the same <u>phase</u> (like photons in laser)

$$\psi = \sqrt{n_s}\, e^{i\phi}$$

$\sqrt{n_s}$ ↑ density of superconducting e^- pairs

ϕ ↑ phase

What does it really mean that all pairs have the same wave function?

- Identical particles
  - All electrons (fermions) are indistinguishable in quantum mechanics.
  - All electron pairs – Cooper pairs – (bosons) are indistinguishable in quantum mechanics.
- When identical bosons "condense", we got a Bose-Einstein condensation. → Superfluid (no charge), Superconductor (charge)

A quantum state with macroscopic coherence

What does it really mean that all pairs have the same wave function?

- All bosons in the same wave functions.
- A crude analogy is the vibrations in a guitar string or an ocean wave or any similar wave phenomenon. (Although such systems are not quite clean enough. Laser is a much better analogy.)
- **A large amplitude in a wave means many quanta of vibrations – many bosons!**
- Can we distinguish one vibration quantum from another when a guitar string vibrates? No. Such a state must be viewed as **collective**.
- Except that in superconductors, these indistinguishable bosons are "tangible stuff" electron pairs – so, it is mind-boggling…

A quantum state with macroscopic coherence (all = one)

So How does it work?

charge flux or current density

$$\vec{j}_0 = \frac{\hbar q}{2im}\left(\psi_0^* \vec{\nabla}\psi_0 - (\vec{\nabla}\psi_0)^* \psi_0\right)$$

$$\vec{j} = \vec{j}_0(\psi_0 \to \psi) - \frac{q^2 \vec{A}}{mc}|\psi|^2$$

Hamiltonian in presence of **A**

$$H = \frac{1}{2m}\left(\vec{p} - \frac{q\vec{A}}{c}\right)^2 + \text{pot.}$$

$\psi \approx \psi_0$ (energy gap, many-body coherence)

$\vec{j}_0 = 0$ $\qquad$ $\vec{j} = -\frac{q^2 \vec{A}}{mc} n_s$

London Equation

$q = -2e,\ m \approx 2m_e$

Cooper Pair

$$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\vec{j}$$

$$-\vec{\nabla}^2 \vec{A} = \frac{4\pi}{c}\vec{j} = -\frac{4\pi q^2 n_s}{mc^2}\vec{A}$$

$\lambda \sim$ a few 100 Å

$$\vec{\nabla}^2 \vec{A} = \frac{\vec{A}}{\lambda^2} \qquad \lambda = \sqrt{\frac{mc^2}{4\pi q^2 n_s}}$$

cgs unit

for SI

$c \to 1$

$4\pi \to \mu_0$

$\vec{B}$ field is screened within the length scale λ

Meissner Effect

Steady state $\quad$ No electrostat. pot.

$$\frac{\partial \vec{j}}{\partial t} = 0 \qquad \vec{E} = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t} \qquad \vec{E} = 0$$

Infinite Conductivity

Two Length Scales and two types of SC

λ = London Penetration Depth

ξ = coherence length $\sim \frac{\hbar v_F}{\Delta}$

Δ : pair binding energy

$\xi \sim$ Pair wavefunction Size

Pippard non-local E & M $\leftarrow$ due to ξ

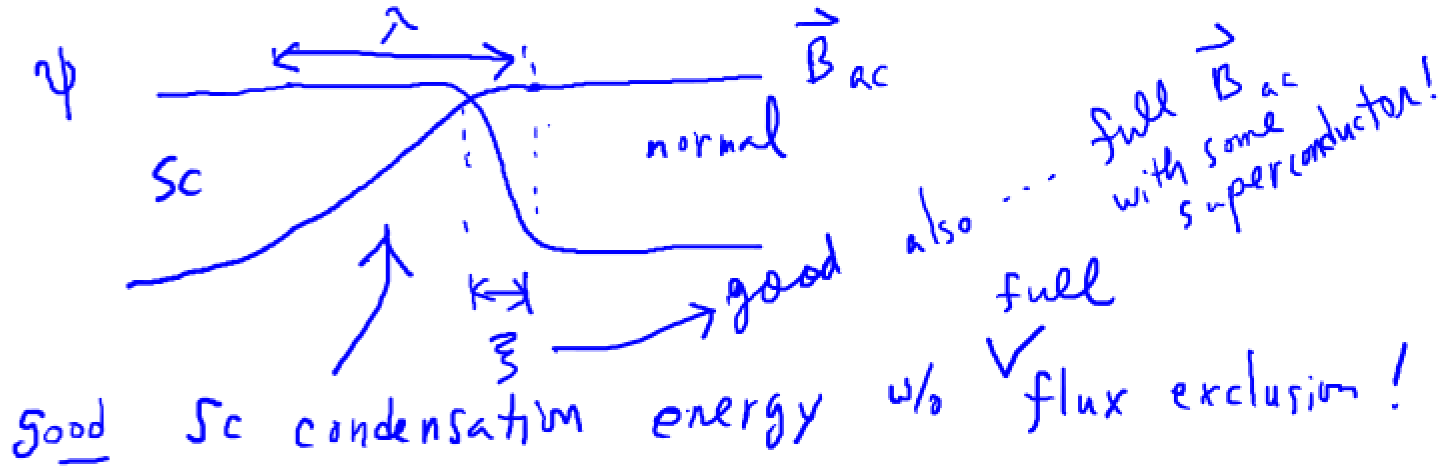
As material becomes impure, λ and ξ change in opposite manners.

Type 1 SC : $\lambda(T=0) \ll \xi$

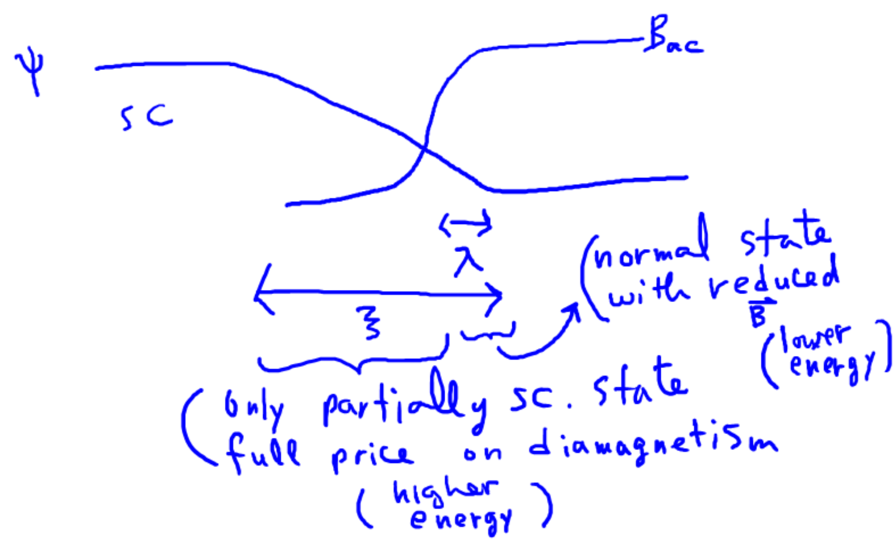
Type 2 SC : $\lambda(T=0) \gg \xi$ (most useful)

Two Length Scales and two types of SC

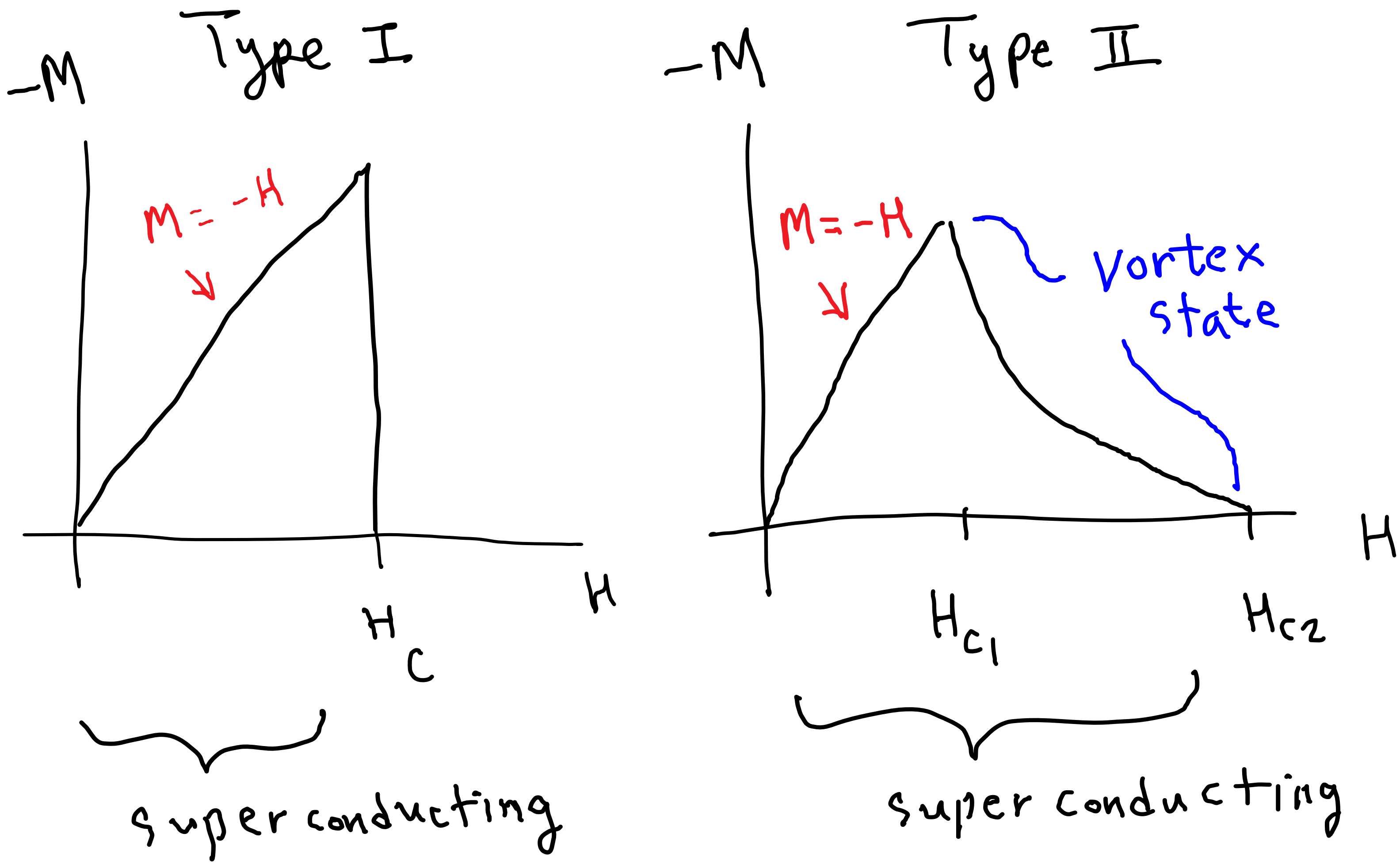
Type II



Type I



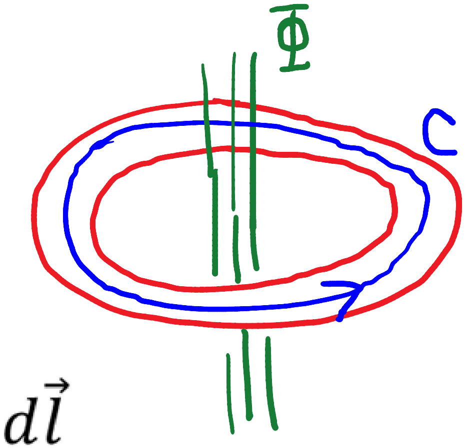
Two types of SC

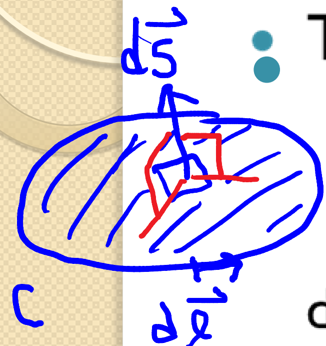


Flux quantization

- The wave function $\psi = \sqrt{n_s}\, e^{i\phi}$ is fine for a linear superconductor. But, how about in general, e.g., for a ring of superconductor? ϕ must be a function of position, $\phi(\vec{r})$.
- Then, the charge flux $\vec{j} = \frac{n_s q}{m}\left(\hbar\vec{\nabla}\phi - q\vec{A}\right)$ (from slide 14; SI unit now).
- In the interior of the ring superconductor, $\vec{j} = 0$ (as $\vec{B} = 0$), and so $q\vec{A} = \hbar\vec{\nabla}\phi$.

Flux quantization





- The magnetic flux is given by

$$\Phi = \int \vec{B} \cdot d\vec{S} = \oint_C \vec{A} \cdot d\vec{l}$$

due to $\vec{B} = \vec{\nabla} \times \vec{A}$ and Stoke's theorem.

- This and $\vec{A} = \frac{\hbar}{q} \vec{\nabla}\phi$ means that $\Phi = \frac{\hbar}{q}(\phi_f - \phi_i)$. But $\phi_f - \phi_i = 2\pi n$ where n is an integer, since the spatial wave function ψ must be single-valued. Thus

$$\Phi = \frac{h}{q} n$$

$$q = -2e \qquad \frac{h}{2e} = 2 \times 10^{-15} \text{ Tesla m}^2$$