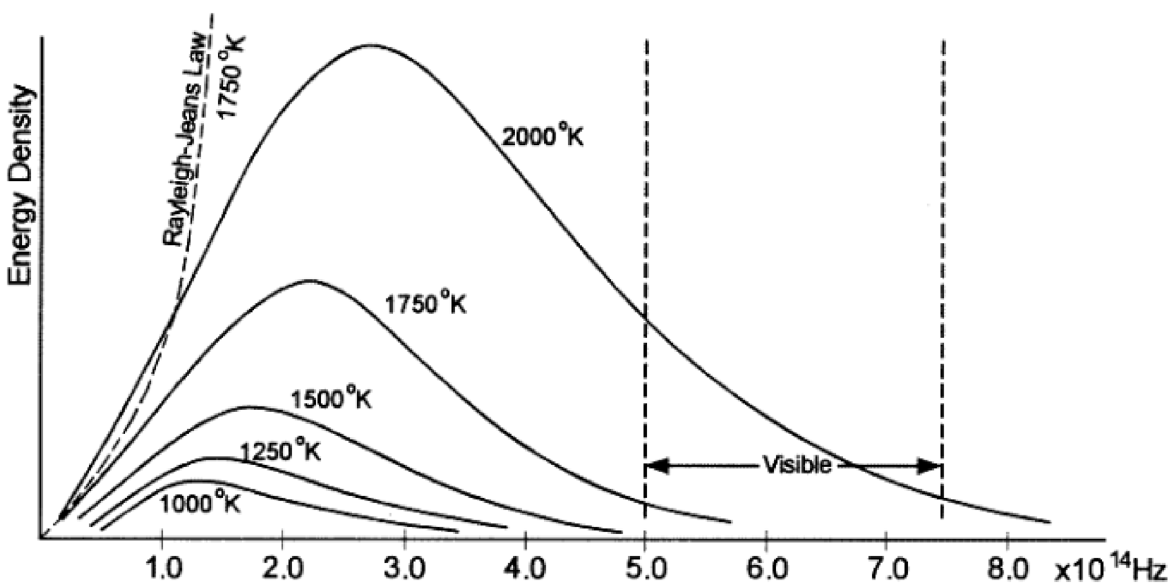
PH102, 2012W, Lecture Notes: February 10, Fri, Class 13

Black Body Radiation

- Every substance emits electromagnetic radiation depending upon the temperature of the substance.
- A "black body" refers to an ideal body that absorbs all radiation incident upon it regardless of frequency and emits the radiation.
- The spectral distribution of energy in the radiation depends only upon the temperature of the body as shown below:

Figure 6.24



- By experimental results, the following two relationships were discovered:
  - Wien's law: the frequency at which the maximum photon energy occurs is proportional to temperature of the black body.
  - Stefen-Boltmann law: The intensity of radiation depends on T^4.
- Planck proposed the idea of light quanta to match the graph above for all frequency ranges.

$$E_n = nh\nu$$

Where n represents the quantum number for the photon where $n = 0,1,2,3 \ldots \infty$, and ν represents frequency. Average Energy $\bar{E}$ can be calculated by using the Botzman expression.

$$\bar{E} = \sum E_n P(E_n) = \frac{\sum_n E_n e^{-\frac{E_n}{k_B T}}}{\sum_n e^{-\frac{E_n}{k_B T}}}$$

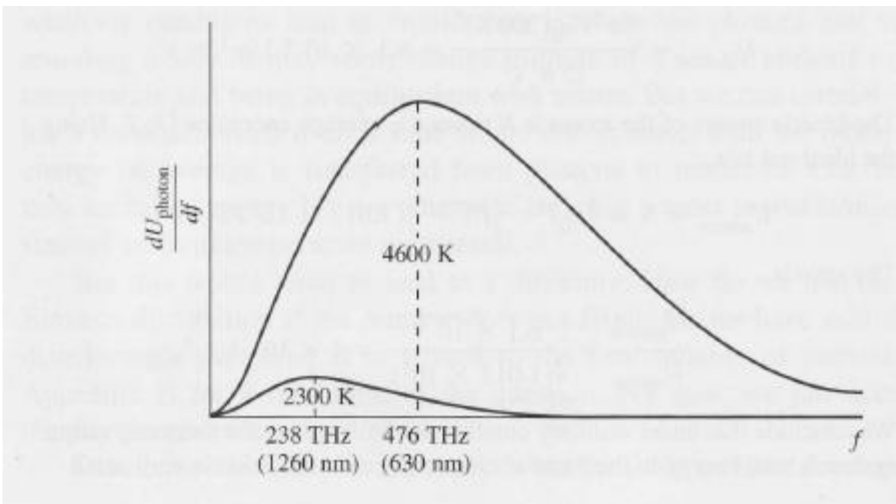
$$\bar{E} = \frac{h\nu}{e^{\frac{h\nu}{k_B T}} - 1}$$

- Density of States: $D(\nu) = \frac{8\pi V}{c^3}\nu^2$ or using $(E = h\nu)$ $D(E) = \frac{8\pi V}{h^3c^3}E^2$
- $E = \int_0^\infty E\mathcal{N}(E)D(E)dE = \int_0^\infty \frac{E}{e^{E/k_BT}-1}\left(\frac{8\pi V}{h^3c^3}E^2\right)dE$

Use $E \equiv k_BTx$

$$= \frac{8\pi V}{h^3c^3}\int_0^\infty (k_BT)^4\frac{x^3}{e^x-1}dx = \frac{8\pi V(k_BT)^4}{h^3c^3}\frac{\pi^4}{15} = \frac{8V\pi^5k_B^4}{h^3c^3}T^4$$

→ This is in agreement with Stephen Boltzmann's law



- Also, the above graph is based on the energy formula, we can see Wien's law results.

$$dE = \frac{h\nu^3}{e^{h\nu/k_BT}-1}\left(\frac{8\pi V}{c^3}\right)d\nu$$

(see 238 THz at 2300K → 476 THz at 4600K).

Compare Energy of Photons at the room temperature with Energy of a one Mole ideal gas:

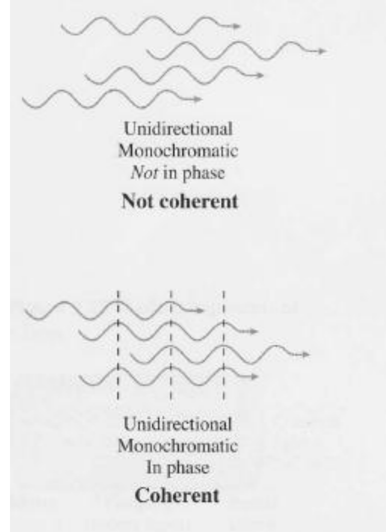
- Energy of 1 mole ideal gas molecules at 273K, 1 atm $= \frac{3}{2}RT = \frac{3}{2}PV = \frac{3}{2}(1.013\ x\ 10^5 Pa)$V
- Energy of photon at 300K = $\int_0^\infty E\mathcal{N}(E)D(E)dE = \int_0^\infty \frac{E}{e^{k_BT}-1}(\frac{8\pi V}{h^3c^3}E^2)dE$

Use $E \equiv k_BTx$

$= \frac{8\pi V}{h^3c^3}\int_0^\infty (k_BT)^4\frac{x^3}{e^x-1}dx = \frac{8\pi V(k_BT)^4}{h^3c^3}\frac{\pi^4}{15} = 6.1\ x\ 10^{-6}\ J/m^3V$

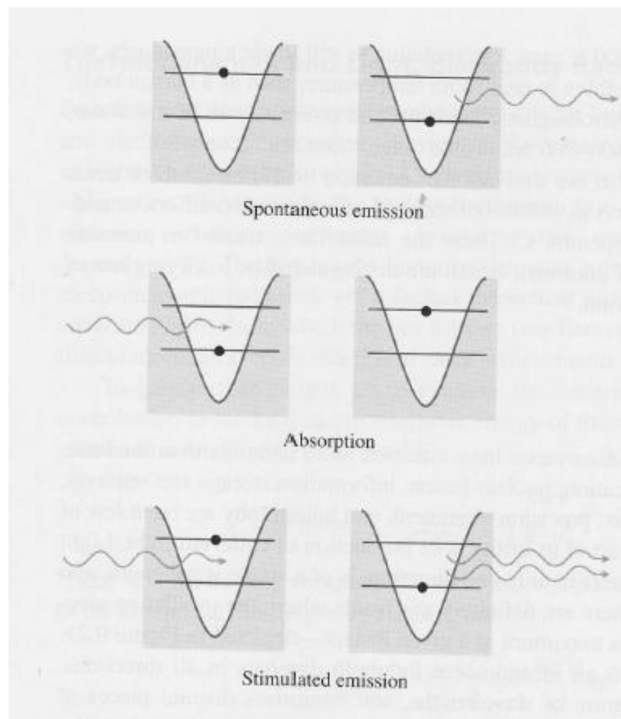
→ is very small compared to energy of 1 mole ideal gas

Figure 9.21 Coherent versus incoherent light.



Laser

- LASER means "Light Amplification by the Stimulated Emission of Radiation"
- Light produced by the Laser is highly coherent. Light can be coherent when it travels in only one direction, is of a single wavelength, and is in phase.
- Spontaneous Emission: Light (Photon) is emitted from the higher energy level (E_2) to the lower energy level (E_1) with the photon energy, $h\nu$, which corresponds to the energy difference ($h\nu = E_2 - E_1$). The rate at which the spontaneous emission occurs from $E_2$to E_1 is proportional to how many particles are at E_2.
$$R_{spo} = A_{spo}N_2$$
- Absorption: Light (Photon) with the energy of $h\nu$ can be absorbed to promote the particles in the lower energy state (E_1) to the higher energy state (E_2). The rate at which the absorption to occur should be proportional to the number of particles at E_1 as well as the number of photons, $Y(\Delta E)$, having energy $h\nu$.
$$R_{abs} = B_{abs}N_1Y(\Delta E)$$
- Stimulated Emission: Similarly, the particles at E_2 can emit photons with $h\nu$ when stimulated by the presence of $h\nu$. The rate at which the stimulated emission occurs is proportional to N_2 as well as as well as the number of photons, $Y(\Delta E)$, having energy $h\nu$.
$$R_{sti} = B_{sti}N_2Y(\Delta E)$$



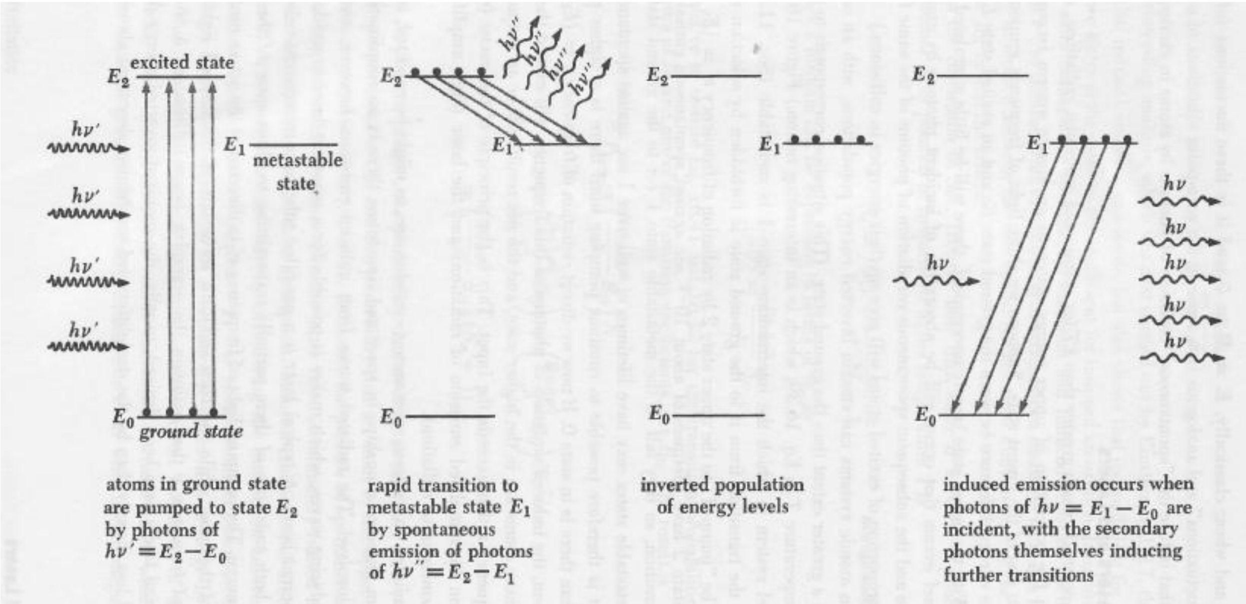
- Principle of detailed balance:
The emission rate = The absorption rate
$$R_{spo} + R_{sti} = R_{abs}$$
$$A_{spo}N_2 + B_{sti}N_2Y(\Delta E) = B_{abs}N_1Y(\Delta E)$$
$$(B_{abs}N_1 - B_{sti}N_2Y)Y(\Delta E) = A_{spo}N_2$$

$$Y(\Delta E) = \frac{A_{spo}N_2}{(B_{abs}N_1 - B_{sti}N_2Y)} = \frac{A_{spo}/B_{abs}}{\frac{N_1}{N_2} - \frac{B_{sti}}{B_{abs}}} = \frac{A_{spo}/B_{abs}}{e^{\Delta E/k_BT} - \frac{B_{sti}}{B_{abs}}}$$

Since $N_1 \propto e^{-\frac{E_1}{k_BT}}$ and $N_2 \propto e^{-\frac{E_2}{k_BT}}$

This formula represents the number of photons with the energy difference of $h\nu$, knowing photon is a boson, then $B_{sti} = B_{abs}$. We can have photon energy of $h\nu$ that can be equally likely to be absorbed by the particles in the lower energy state and to cause the stimulated emission of the particles in the higher energy state.

- Population Inversion: Increase the number of particles at the higher energy state. We cannot simply increase the number of particles at the higher energy state because the occupation number drops exponentially as energy increases.
- Use of a metastable state, E_1.
  - Through optical pumping with photons with energy $h\nu'(= E_2 - E_0)$leads the equilibrium state where $N_0 = N_2$
  - The particles at E_2has a preferential drop to the E_1state than the E_0state, emitting $h\nu'' = E_2 - E_1$, making $N_2 > N_1$.
  - When photons with $h\nu = E_1 - E_0$ can induce stimulated emission of coherent photons with $h\nu$.



- Laser examples:
  - Ruby lasers: three level with optical pumping produced 693.4 nm light.
  - Four level neodymium glass lasers produce 1060 nm.