

Lecture Notes: January 18, Wed, Class 4

Hydrogen Atom Models and Spectral Lines

Objectives:

- (Cont.) Express the time-independent Schrodinger Equation for a particle in a 3-dimensional infinite well situation using (x, y, z) coordinates. Describe quantized energy states and wave functions. Understand energy degeneracies and splits.
- (HW) Clarification on potential barrier wave functions.
- Qualitatively understand relationships between various hydrogen atom models and spectral lines.

Hydrogen Atom Spectral Lines

The electron in the hydrogen atom is allowed to have the following energy levels:

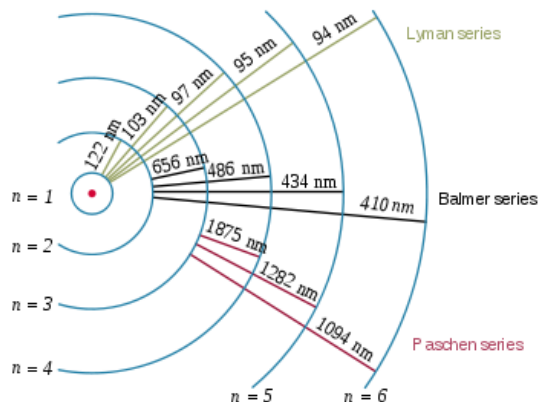
$$E_n = -\frac{me^4}{2(4\pi\epsilon_0)^2\hbar^2} \frac{1}{n^2} = -\frac{e^2}{8\pi\epsilon_0} \frac{me^2}{4\pi\epsilon_0\hbar^2} \frac{1}{n^2} = -\frac{e^2}{8\pi\epsilon_0 a_0} \frac{1}{n^2} = (-13.6 \text{ eV}) \frac{1}{n^2}$$

$$\text{Where } a_0(\text{Bohr Radius}) = \frac{me^2}{4\pi\epsilon_0\hbar^2} = 0.0529 \text{ nm} = 0.529 \text{ \AA}$$

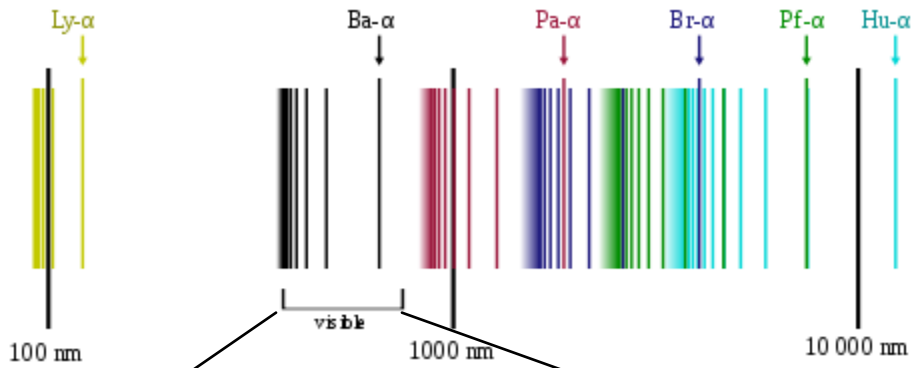
If the electron in the hydrogen atom occupies at a higher energy state, then the electron can transition from the higher to the lower energy state by emitting the photon of which energy equals the difference between the two energy states.

$$E_{\text{photon}} = \frac{hc}{\lambda} = E_{\text{initial}} - E_{\text{final}} = -\frac{me^4}{2(4\pi\epsilon_0)^2\hbar^2} \left(\frac{1}{n_{\text{final}}^2} - \frac{1}{n_{\text{initial}}^2} \right)$$

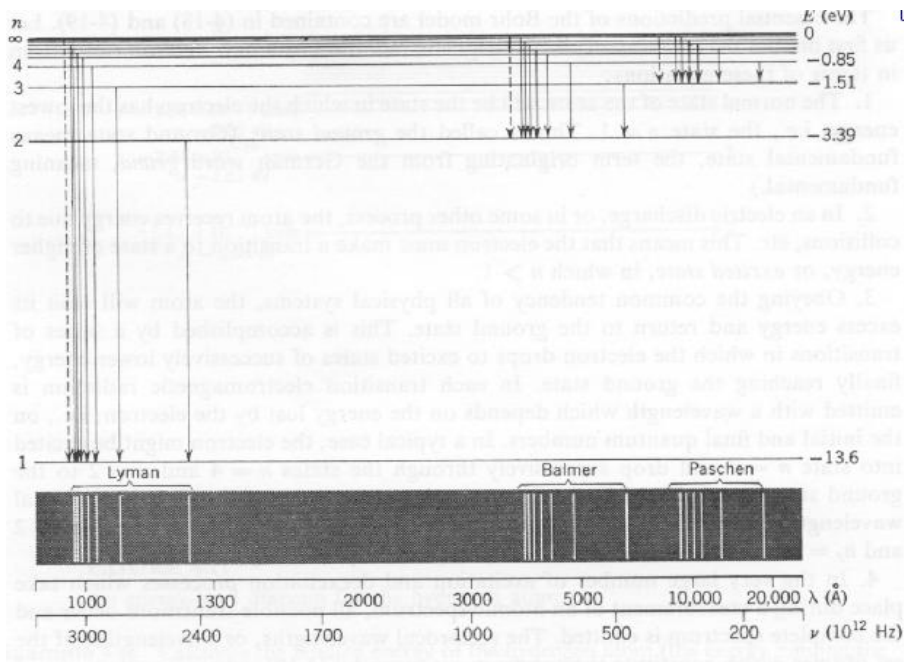
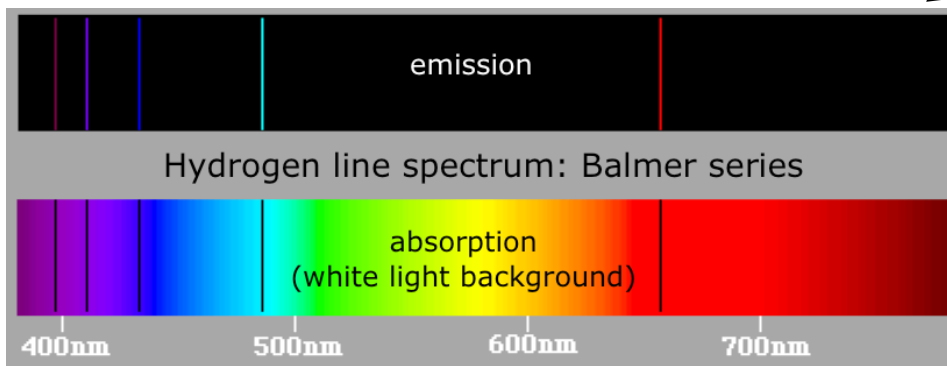
$$\frac{1}{\lambda} = \frac{me^4}{2(4\pi\epsilon_0)^2\hbar^2 hc} \left(\frac{1}{n_{\text{final}}^2} - \frac{1}{n_{\text{initial}}^2} \right) = 1.097 \times 10^7 \text{ m}^{-1} \left(\frac{1}{n_{\text{final}}^2} - \frac{1}{n_{\text{initial}}^2} \right)$$



- When $n_{\text{final}} = 1$, *Lyman Series* (discovered between 1905-1914)
- When $n_{\text{final}} = 2$, *Balmer Series* (in 1885)
- When $n_{\text{final}} = 3$, *Pachen Series* (in 1908)
- When $n_{\text{final}} = 4$, *Brackett Series* (in 1922)
- When $n_{\text{final}} = 5$, *Pfund Series* (in 1924)
- When $n_{\text{final}} = 6$, *Humphreys Series* (in 1953)



Spectral lines in a logarithmic scale (only the Balmer series is visible).



From quantummechanics.ucsd.edu

Lyman Series ($n_{final} = 1$)

$n_{initial}$	λ (nm)
2	122
3	103
4	97.3
5	95.0
6	93.8
∞	91.2

Balmer Series ($n_{final} = 2$)

$n_{initial}$	λ (nm)
3	656
4	486
5	434
6	410
7	397
∞	365

Pachen Series ($n_{final} = 3$)

$n_{initial}$	λ (nm)
4	1870
5	1280
6	1090
7	1020
8	954
∞	820